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TITLE	MULTIDIMENSIONAL INTEGRATION BY GAUSSIAN QUADRATURE
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MULTIDIMENSIONAL INTEGRATION BY GAUSSIAN QUADRATURE

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I. INTRODUCTION

The user is referred to standard references on numerical methods for detailed discussion of Gaussian quadrature. In brief, an integral of the form

$$\int_{u_a}^{u_b} \int_{u_b}^{v_b} \ldots\ldots F(u,v, \ldots\ldots) \; dudv \ldots\ldots$$

may be approximated by a sum

$$\sum_{i=1}^{n} w_i \sum_{j=1}^{n} w_j \ldots\ldots \quad F(u_i, v_j, \ldots\ldots)$$

where the value q_i [u_i, v_j, etc] depends on q_a, q_b, and i;

with both the weights w_i and the values q_i so chosen as to duplicate the highest possible order polynomial. By evaluating the function at n points, one can obtain the exact result for integration of a power series of 2n terms. This procedure is thus very rapid, and well matched to FOCAL, which (being an interpretive language) is relatively slow in execution.

The three sections 13-15 constitute a Gaussian quadrature package, to be inserted into programs as desired. Section 15 contains the main subprogram, with 14 providing a gaussian-coefficient input segment and 13 providing a point at which the user can introduce functions defining integration limits. The routine can do any order of integration that storage will allow.

II. USAGE

The segments are loaded as part of a user program. The first reference to the package must be a DO 14, allowing input of the Gaussian coefficients. Coefficients for three and five points are provided with the program.

Integration is performed to defining the function to be integrated as Y in section 10. Y must be calculated in terms of values of X(I) produced by section 15. The master program must then define the order or dimension of the integration (IM or IMAX), set upper and lower integration limits (XO(T) and XM(I)) if these are constant, and enter the integration subprogram by a DO 15. If limits are functions of some of the variables X(I), equations defining these limits should be inserted in a section 13, replacing the simple RETURN instruction. Upon return from section 15, the desired integral will be in S(1).

III. LIMITATIONS

This routine works well for functions that can be approximated with reasonable accuracy by polynomials of reasonable order. Functions with discontinuities in value or in slope may best be integrated piecewise.

For programming economy, only odd values of n are allowed, and n must be the same in each dimension.

B. The volume of one octant of a sphere. This example shows the use of changing limits in section 13. This shows, also, a case in which Gaussian Quadrature is particularly poor, because the function desired cannot be well approximated by a polynomial.

```
C-8K FOCAL @1969

01.01 C VOLUME OF A SPHERE-OCTANT
01.10 SET IMAX=2; SET XM(1)=1; DO 14; FOR I=1,2; SET XO(I)=0
01.20 DO 15; TYPE !,%,"VOLUME",S(1);Q

10.10 SET Y=FSQT(1-X(1)↑2-X(2)↑2)

13.10 IF (I-2) 13.2;SET XM(2)=FSQT(1-X(1)↑2)
13.20 R

14.10 A "COEF. FOR GAUSSIAN INTEG.; PTS",JM
14.20 S JM=FITR(JM/2)
14.30 F J=0,JM; A XG(J),W(J)

15.01 S I=1
15.10 S J(I)=-JM; S S(I)=0; DO 13
15.12 S A(I)=(XM(I)-XO(I))/2; S B(I)=A(I)+XO(I)
15.20 S JP=FABS(J(I))
15.22 S X(I)=A(I)*XG(JP)*FSGN(J(I))+B(I)
15.30 IF (I-IMAX) 15.31, 15.4
15.31 S I=I+1; GT 15.1
15.40 DO 10; S S(IM+1)=Y
15.50 S S(I)=S(I)+W(JP)*S(I+1)*A(I); S S(I+1)=0
15.60 IF (J(I)-JM) 15.61; S I=I-1; GT 15.7
15.61 S J(I)=J(I)+1; GT 15.2
15.70 IF (I) 15.8, 15.8; DO 15.2; GT 15.5
15.80 R
*GO
COEF. FOR GAUSSIAN INTEG.; PTS:3
:.0000000 :.8888888
:.7745967 :.5555555
VOLUME= 0.5260120002E+00*
*GO
COEF. FOR GAUSSIAN INTEG.; PTS:5
:.0000000 :.5688888
:.5384693 :.4786287
:.9061798 :.2369269
VOLUME= 0.5241969663E+00*
*GO
COEF. FOR GAUSSIAN INTEG.; PTS:7
:.0000000000 :.4179591836
:.4058451513 :.3818300505
:.7415311856 :.2797053915
:.9491079123 :.1294849662
VOLUME= 0.5238319460E+00*
*
*TYPE (4/3)*3.1415926/8
= 0.5235987667E+00*
```

V. EXAMPLES

A. A simple sine integration in three dimensions.

```
C-8K FOCAL @1969

01.10 DO 14
01.20 SET PI=3.1415926;   SET IMAX=3
01.30 FOR I=1,3;SET XM(I)=PI/2;SET XO=0
01.40 DO 15
01.50 TYPE %9.09,S(1); QUIT

10.10 SET Y=FSIN(X(1))*FSIN(X(2))*FSIN(X(3))

13.10 R

14.10 A "COEF. FOR GAUSSIAN INTEG.; PTS",JM
14.20 S JM=FITR(JM/2)
14.30 F J=0,JM; A XG(J),W(J)

15.01 S I=1
15.10 S J(I)=-JM; S S(I)=0; DO 13
15.12 S A(I)=(XM(I)-XO(I))/2; S B(I)=A(I)+XO(I)
15.20 S JP=FABS(J(I))
15.22 S X(I)=A(I)*XG(JP)*FSGN(J(I))+B(I)
15.30 IF (I-IMAX) 15.31, 15.4
15.31 S I=I+1; GT 15.1
15.40 DO 10; S S(IM+1)=Y
15.50 S S(I)=S(I)+W(JP)*S(I+1)*A(I); S S(I+1)=0
15.60 IF (J(I)-JM) 15.61; S I=I-1; GT 15.7
15.61 S J(I)=J(I)+1; GT 15.2
15.70 IF (I) 15.8, 15.8; DO 15.2; GT 15.5
15.80 R
*GO
COEF. FOR GAUSSIAN INTEG.; PTS:3
:.0000000 :.8888888
:.7745967 :.5555555
= 1.00002524*
*GO
COEF. FOR GAUSSIAN INTEG.; PTS:5
:.0000000 :.5688888
:.5384693 :.4786287
:.9061798 :.2369269
= 1.00000135*
*GO
COEF. FOR GAUSSIAN INTEG.; PTS:7
:.0000000000 :.4179591836
:.4058451513 :.3818300505
:.7415311856 :.2797053915
:.9491079123 :.1294849662
= 1.00000134*
*
*C -- ERROR OF .00000135 IS PROBABLY ROUNDOFF IN FOCAL TRIG
*C    FUNCTIONS.
```