

# DECUS

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DECUS NO.	FOCAL8-50
TITLE	FOCAL VERSION OF RC ACTIVE FILTER
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# Get something extra in filter design.

One BASIC program works for Butterworth and Chebyshev low-pass or high-pass RC-active circuits.

A time-shared computer program can do much more than free the engineer from the tedium of routine calculations in filter design.<sup>1</sup> A single program in BASIC,<sup>2</sup> derived from two fundamental equations, can be used to design Butterworth or Chebyshev filters, and either low-pass or high-pass versions of each.

The program, written in a language resembling simple English, determines the component values for  $N$ -pole filters. The design uses two-pole active sections with only Rs and Cs, no Ls, as the basic building blocks for higher-order filters.  $N$ -pole filters are created by cascading  $N/2$  two-pole sections. The R and C values in each section are selected to achieve the desired filter response.

Low-pass and high-pass sections<sup>3,4,5</sup> used in the filters are shown in Fig. 1. Two capacitors, two resistors and a unity-gain active element (Table 1) serve to synthesize a complex pair of poles in the filter characteristic.

The filters described in this article are relatively inexpensive. They may be built as either discrete circuits or hybrid microcircuits and for either commercial or military use.

As hybrid microcircuit designs they possess the following advantages:

- Since they use no Ls, the resulting circuit is potentially smaller, more stable and has a higher Q at low frequencies than passive LC designs.

- The Cs can be chosen as standard values. Even though the Rs are non-standard, they are relatively easy to obtain.

- The frequency response can be adjusted by varying only the Rs. One filter can therefore be readily tuned into phase track with another, or trimmed to a given specification.

In addition this design approach:

- Uses a minimum of Rs and Cs to synthesize a two-pole function.

- Requires only one unity-gain active element for each filter section.

- Has low sensitivity to parameter changes.

Several of the many possible types of unity-gain active elements are shown in Table 1. The most important figure of merit for these voltage

follower elements is their current gain  $\beta$  because accurate filter synthesis requires a high input impedance and low output impedance. The equations used to calculate the filter component values assume a perfect active element,  $\beta = \infty$ . In practice the active elements are imperfect, especially at higher frequencies. Finite input impedance causes insertion loss and frequency response distortion; non-zero output impedance causes reduced stop-band attenuation; and variations from unity-gain change the resonant response of the section.

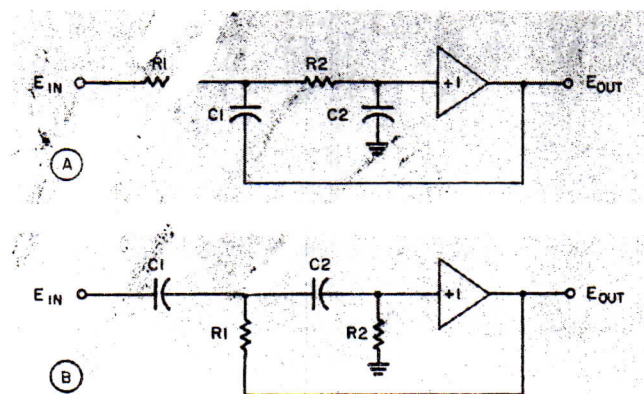
In view of these effects, it makes little sense to use 1% components to obtain a precise filter response, unless the input impedance of the active element is greater than  $100R_2$  and the output impedance is less than  $R_1/100$ . It is also senseless to seek high stop-band attenuation in a frequency region where  $\beta$  is significantly decreased.

## Align sections by adjusting only resistors

Component values (Fig. 1) for the basic low-pass or high-pass filter sections are computed from the equations for the pole locations ( $\sigma_i + j\omega_i$ ) of the normalized functions:

$$\sigma_i = X1 \cos (P_i)$$

$$j\omega_i = jX2 \sin (P_i).$$



1. Two-pole circuit sections serve as basic building blocks that are cascaded to form multiple-pole filters, low-pass section above, and high-pass section, below. The component values in each section are computer-selected (to obtain either a Butterworth or Chebyshev response).

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where  $X_1 = 1$ ,  $X_2 = 1$ , for Butterworth filters, and  $X_1$  and  $X_2$  depend on the passband ripple and number of poles for Chebyshev filters. The equations for the component values are shown in Table 1.

The over-all filter design is not limited to a particular ratio of component values. Each two-pole section may therefore be independently aligned by adjusting only the two resistor values. Three cases occur:

1. When the component values of a section are fairly close to the design values, the filter alignment can be improved by nearly an order of magnitude by adjusting only one of the two resistors so that the desired response is obtained at the cut frequency. For thick-film or potted sections, the adjustment may be made externally by adding a series trim resistor either to  $R_1$  of the low-pass circuit or to  $R_2$  of the high-pass circuit (Fig. 1).

2. When both C values are out of tolerance by comparable large percentages, the section may be aligned by a two-step procedure. First, impedance-scale the section by off-adjusting the two resistors by the same percentage as the capacitors but in the opposite direction. This will improve the frequency response of the section. It will modify the section impedance level to accommodate the varied C values. Second, trim the response of the section at the cut frequency by adjusting one of the two resistors, as previously described. It is not advisable to trim both resistors since their effects on the frequency response are interdependent.

3. When the two C values are out of tolerance by different large percentages, the section response may best be improved by off-adjusting the Rs to recomputed values. The revised R values are obtained by rerunning the computer program with the actual C values inserted in place of the nominal ones. One of the two resistors may then be trimmed, if desired.

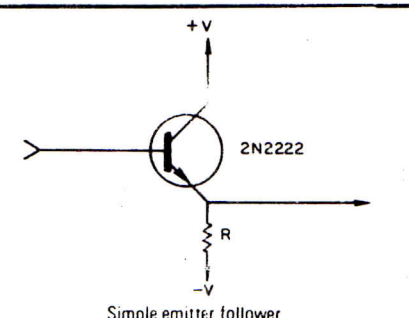
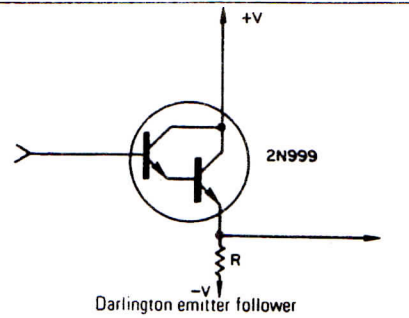
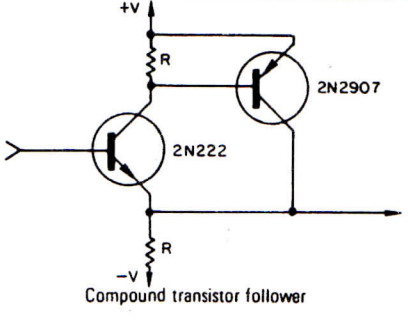
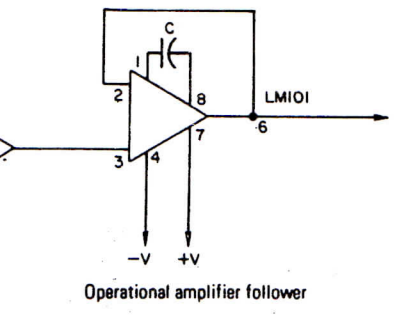
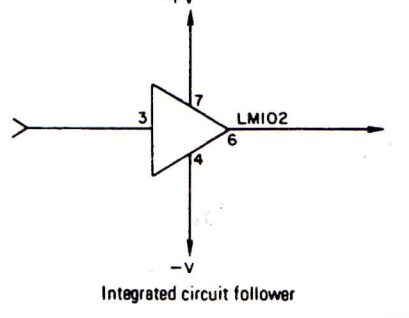
### Sensitivity influences filter response

If the circuit component values are out of tolerance—due to initial selection error, environmental variation or aging—the filter response will vary from nominal. The variation of filter gain  $\Delta G/G$  with component value variation  $\Delta V/V$  is determined by the sensitivity factor  $S$ :

$$\Delta G/G = S \times \Delta V/V.$$

If  $S = 1$ , the equation shows that a component variation of 1% is equivalent to  $20 \log_{10} (1-0.01)$ , resulting in a gain variation of only 1 dB. For a large value of  $S$ —say,  $S = 100$ —the same equation shows that a component variation of 1% is equivalent to  $100 \times (0.01) = 1$  (or a 100% increase). As the component varies by 1%,  $G$

**Table 1. Unity gain active elements**

Circuit element	Approximate current gain ( $\beta$ ) (at low frequencies)
 <p>Simple emitter follower</p>	100
 <p>Darlington emitter follower</p>	4000
 <p>Compound transistor follower</p>	4000
 <p>Operational amplifier follower</p>	>100,000
 <p>Integrated circuit follower</p>	>100,000

**Table 2. Filter component value formulas\***

Low-pass section		High-pass section	
Formula	Eq. Nos. (see box)	Formula	Eq. Nos. (see box)
$C_1 = C$	14a	$C_1 = C$	18a
$C_2 = C/M$	14b	$C_2 = C$	18b
$R_1 = \frac{-\sigma_i}{2\pi F C} \left[ 1 + \left( 1 - \frac{A_i}{\sigma_i^2 M} \right)^{\frac{1}{2}} \right]$	15a	$R_1 = \frac{-\sigma_i}{2\pi FC}$	19a
$R_1 = \frac{-\sigma_i M}{2\pi F A_i C} \left[ 1 - \left( 1 - \frac{A_i}{\sigma_i^2 M} \right)^{\frac{1}{2}} \right]$	15b	$R_2 = \frac{-A_i}{2\pi F \sigma_i C}$	19b
$M \geq A_i / \sigma_i^2$	16		

\*Resistor values are scaled to cut frequency F.

increases 100%, or from G to 2G, a gain variation of 6 dB ( $20 \log_{10} 2 = 6$  dB).

Since a sensitivity factor is associated with each component, the worst-case variation for the complete section occurs when each component has a maximum error in an additive direction.

The sensitivity factors for components in the basic active filter sections (Fig. 1) vary with frequency and section Q. For the worst-case frequency, the sensitivity, S, for each of the R and

C components (Fig. 1) is:

$\frac{Q}{1}$	$\frac{S}{0.01}$
100	0.3

and for variations in gain of the active element is

$\frac{Q}{1}$	$\frac{S}{0.02}$
5	1.0
8	100.0

The high-gain sensitivity factor is not harm-

```

0 DATA 2,4,1,2.4,10
10
20 REM RC ACTIVE FILTER PROGRAM BY R. KINCAID AND F. SHIRLEY
30 REM SANDERS ASSOC. INC, WASHUA, N.H., 11/3/68
40 REM LINE # SHOULD BE: 0 DATA T,N,F,R,C -- WHERE T IS THE TYPE
50 REM (1=LOWPASS, 2=HIGHPASS); N IS THE NO. OF POLES (MUST BE EVEN);
60 REM F IS THE CUT FREQ IN KHZ; R IS THE PASSBAND RIPPLE IN DB (0
70 REM FOR BUTTERWORTH); AND C IS THE MAX C IN NANOFARADS.
80
90 READ T,N,F,R,C
100 LET Q0=6.28318531
110 LET Q1=2.30258509
120 DIM F(37),G(37),C(25)
130 PRINT
140 PRINT
150 PRINT
160 PRINT
170 GOSUB 2020
180
190 REM "1" ITERATION LOOP -- CALCULATIONS FOR EACH 2-POLE SECTION
200 FOR I=1 TO N/2
210 IF R>0 THEN 290
220
230 REM BUTTERWORTH CIRCLE
240 LET X1=1
250 LET X2=1
260 GO TO 350
270
280 REM CHEBYSHEV ELLIPSE
290 LET E=SQR(10*(R/10)-1)
300 LET D=(1/E+SQR(1/E+1))*(1/N)
310 LET X1=(D-1/D)/2
320 LET X2=(D+1/D)/2
330
340 REM ROOT PAIR LOCATIONS
350 LET P=Q0/4+Q0/4/N*(2*1-1)
360 LET S=X1+Q0*(P)
370 LET W=X2+SIN(P)
380 LET A=S+W*W
390 LET RB=-S/Q0/F/C*IE3
400 LET C1=C
410 IF T=1 THEN 550
420
430 REM LOWPASS C2 VALUE
440 IF T=1 THEN 500
450 DATA 6, 1.0, 1.5, 2.2, 3.3, 4.7, 6.8
460 READ K6
470 FOR K=1 TO K6
480 READ C(K)
490 NEXT K

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500 LET N2=INT(LOG(C)/Q1)
510 FOR N1=-1 TO 99
520 FOR K=N6 TO 1 STEP -1
530 LET C2=C(K)*10*(N2-N1)
540 LET R=C/C2
550 IF M>=R/5/S THEN 600
560 NEXT K
570 NEXT N1
580
590 REM LOWPASS R1,R2 VALUES
600 LET R1=R0*M/A*(1+SQR(1-A/S/S/M))
610 LET R2=R0*M/A*(1-SQR(1-A/S/S/M))
620 GO TO 700
630
640 REM HIGHPASS C2,R1,R2 VALUES
650 LET C2=C
660 LET R1=R0
670 LET R2=R0*A/S/S
680
690 REM PRINTOUT
700 IF I=1 THEN 720
710 PRINT "COMPONENT VALUES (C IN NANOFARADS, R IN KILOHMS):"
720 PRINT
730 PRINT "SECTION",I
740 PRINT "C1=",C1,"R1=",R1
750 PRINT "C2=",C2,"R2=",R2
760
770 REM FREQ AND GAIN VALUES FOR GRAPH
780 FOR K=0 TO 36
790 IF I=1 THEN 870
800 IF T=1 THEN 830
810 LET F(K)=F/25*(1+K)
820 GO TO 840
830 LET F(K)=F*25/(37-K)
840 LET K7=2-INT(LOG(F(K))/Q1)
850 LET F(K)=INT(F(K)*10*(K7+.5))/10*K7
860 LET G(K)=0
870 IF T=1 THEN 900
880 LET G=(1-R1/C1/R2/C2*(Q0*F(K))+2/IE6)*2+((R1+R2)*C2*Q0*F(K)/IE3)+2
890 GO TO 910
900 LET G=(1-1/R1/C1/R2/C2/(Q0*F(K))+2/IE6)*2+(2/R2/C1/Q0/F(K)*IE3)+2
910 LET G(K)=G(K)-10*LOG(G)/Q1
920 NEXT K
930 NEXT I
940 GOSUB 4020
950 STOP
2000
2010 REM HEADING SUBROUTINE
2020 IF R=0 THEN 2030
2030 PRINT " BUTTERWORTH "

```



**Table 3. BASIC commands**

Type	Word	Function
Nonexecutable	REM	Allows the insertion of remarks in the program listing
	DIM	Reserves extra memory room for large variable arrays
	DATA	Stores numerical data to be used in the problem solution
Input/Output	READ	Obtains numerical data from DATA statements
	PRINT	Types output statements and numerical answers
Computational	LET	Computes variable values according to algebraic formulas
Sequencing	GO TO	Alters the normal order of computation
	IF...THEN	Conditionally alters the order of computation
	FOR...TO NEXT	Causes the intervening commands to be repeated several times
	GO SUB RETURN	Routes computation to and from a subroutine (subsection) of the program
Termination	STOP	Stops computation (at any point in the program)
	END	Stops computation (this must be the last sequential command in a program)

**Table 5. Variables used in program**

Name	Definition
T	Type of filter (1 = Low-pass, 2 = High-pass)
N	Number of poles
F	Cut frequency (in kHz)
R	Chebyshev passband ripple (in dB)
C	Maximum circuit capacitance (in nanofarads)
$Q\phi$	$2\pi$ (phase conversion constant from radians to degrees)
Q1	$1n\ 10$ (gain conversion constant from natural logs to common logs)
i	Iteration index ( $i = 1$ to $N/2$ ) for the 2-pole sections
F (37)	Frequency values (independent variable)
G (37)	Gain values (dependent variable)
C (25)	Standard capacitance values per decade
X1	Minor Chebyshev ellipse radius
X2	Major Chebyshev ellipse radius
R	Chebyshev ripple factor
D	Intermediate Chebyshev parameter
P	Root location phase angle
S	Real component ( $\sigma$ ) of pole location
W	Imaginary component ( $\omega$ ) of pole location
A	Squared magnitude of pole location
$R\phi$	Nominal resistance level
R1,R2,C1,C2	Component values (kilohms and nanofarads)
M	Ratio of C1/C2 in low-pass sections
K6	Number of standard capacitance values per decade
K,N1,I1	Iteration loop indices
N2,K7	Normalization constants
G,I2	Intermediate variables used in graph routine

**Table 4. Program block outline**

Line numbers	What is accomplished
0 - 90	Data input and instructions to user
100 - 120	Constant definition and variable dimensioning
130 - 170	Heading print-out (using subroutine 2000 - 2230)
180 - 670	Calculations for each 2-pole section
220 - 410	Calculation of root pair locations
420 - 620	Calculation of low-pass component values
630 - 670	Calculation of high-pass component values
680 - 750	Component value print-out
770 - 930	Selection of frequency values and calculation of gain values for graph
940 - 950	Graph print-out (using subroutine 4000 - 4440)
2000 - 2230	Print subroutine for page headings
4000 - 4440	Print subroutine for graph

a passband ripple of 2.4 dB and a maximum C value of 10 nF.

To solve the first filter design, we must retype line 0 as follows:

0 DATA 1, 6, 1, 0, 33.

The remark ("REM") statements in Fig. 2 explain how the desired filter specifications are entered into the program (see Table 4). The variables used are listed in Table 5. When the program is run with this "DATA" line, a two-page print-out (Fig. 3) is generated. Each page begins with a descriptive heading. Figure 3A shows the first page, which gives the component values, and Fig. 3B the second page, containing a graph of gain versus frequency.

For low-pass designs, the  $C_1$  value in each of the sections is set equal to the maximum C value specified in the DATA line. In this case  $C_1$  is 33 nF in each of the three sections. The  $C_2$  value is then chosen as the largest value that can be realized from a standard decade list of C values

NUMBER OF POLES = 6  
3-DB FREQ (KHZ) = 1

COMPONENT VALUES (C IN NANOFARADS, R IN KILOHMS):

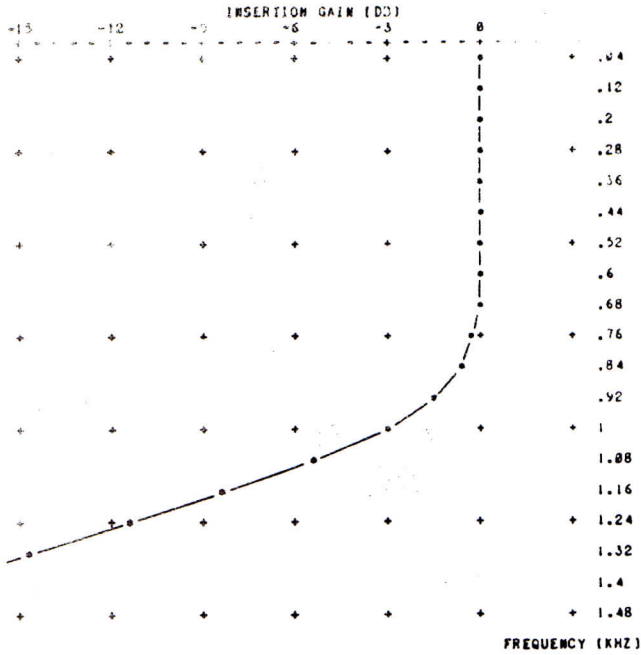
SECTION 1  
C1 = 33 R1 = 20.8192  
C2 = 2.2 R2 = 17.4284

SECTION 2  
C1 = 33 R1 = 9.76477  
C2 = 15 R2 = 5.24051

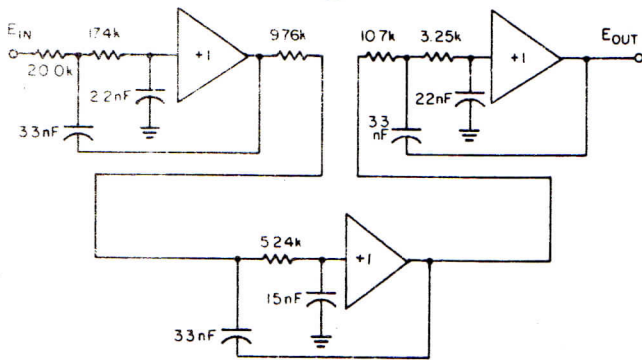
SECTION 3  
C1 = 33 R1 = 10.7214  
C2 = 22 R2 = 3.25427

(A)

NUMBER OF POLES = 6  
3-DB FREQ (KHZ) = 1



(B)



(C)

3. Computer printout of component values (A) and frequency response curve (B) for six-pole Butterworth lowpass filter (C) is shown above.

(contained in line 450 of the program, Fig. 2). The list may be changed, as required, to reflect available C values. The first number in the list (in this case, 6) is the number of values in the list. The following numbers, are the C values

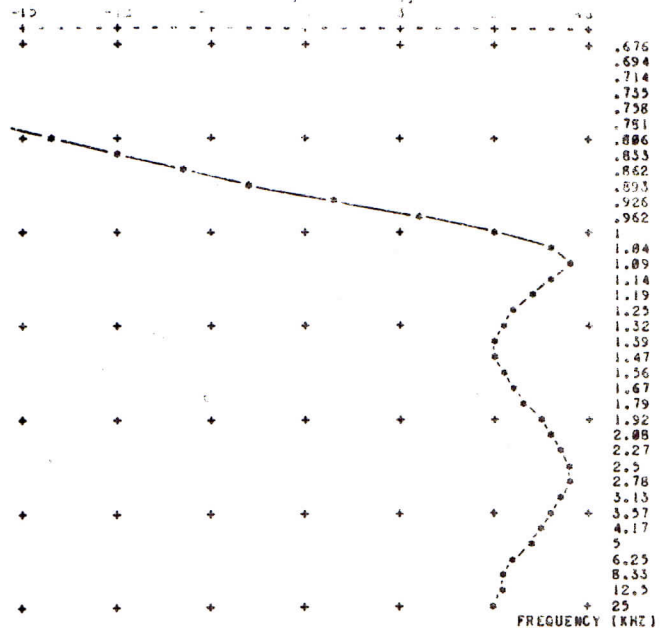
NUMBER OF POLES = 4  
CUT FREQ (KHZ) = 1  
PASSBAND RIPPLE (DB) = 2.4

COMPONENT VALUES (C IN NANOFARADS, R IN KILOHMS):

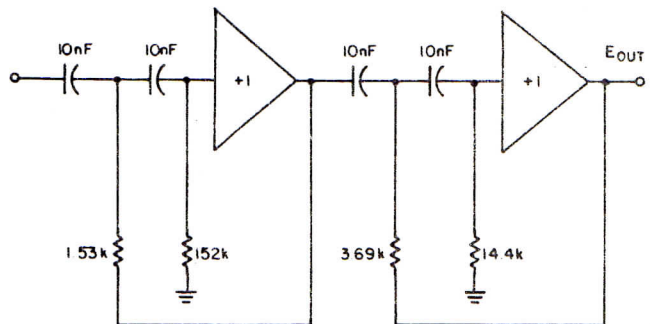
SECTION 1  
C1 = 10 R1 = 1.52754  
C2 = 10 R2 = 151.987

SECTION 2  
C1 = 10 R1 = 5.68733  
C2 = 10 R2 = 14.3802

NUMBER OF POLES = 4  
CUT FREQ (KHZ) = 1  
PASSBAND RIPPLE (DB) = 2.4



(B)



(C)

4. Computer printout of component values (A) and frequency response (B) for four-pole Chebyshev high-pass filter (C) is shown above.

arranged in increasing order of magnitude. In the print-out (Fig. 3A) the values of C<sub>2</sub> for the three sections are 2.2 nF, 15 nF and 22 nF. The R values, which are computed from these C values and the filter specifications, are non-

## Deriving the fundamental filter equations

Modern filter design theory is based upon an analysis of the filter transfer function, the ratio of output to input in the complex frequency plane, the  $s$ -plane. Attention is centered on the location of the "poles" of the transfer function. The poles are the values of  $s$  for which the denominator of the transfer function vanishes, or is equal to zero.

The poles of the normalized Butterworth low-pass function are equally spaced on a unit circle centered at the origin of the  $s$ -plane.<sup>9, 12</sup> For an  $N$ -pole function, the phase angles,  $P_i$ , of the pole locations are:

$$P_i = \frac{\pi}{2} + \frac{\pi}{2N} (2i - 1), \quad i = 1 \text{ to } N. \quad (1)$$

The real ( $\sigma_i$ ) and imaginary ( $j\omega_i$ ) Cartesian coordinates of the pole locations are therefore:

$$\sigma_i = \cos(P_i) \quad (2a)$$

$$j\omega_i = j \sin(P_i) \quad (2b)$$

The poles of the normalized Chebyshev low-pass function are unequally spaced on an ellipse centered at the origin of the  $s$ -plane.<sup>9, 12</sup> The minor radius  $X1$  of the ellipse is parallel to the real ( $\sigma$ ) axis, and the major radius  $X2$  is parallel to the imaginary ( $j\omega$ ) axis of the  $s$ -plane. The values of  $X1$  and  $X2$  depend on the peak-to-peak ripple  $R$  (in dB) of the voltage waveform in the passband, and on the number of poles,  $N$ . It is convenient to define a ripple factor,  $\epsilon$ , in terms of  $R$  according to the following convention:

$$\epsilon \triangleq (10^{R/10} - 1)^{1/2}. \quad (3)$$

If we use this definition then  $X1$  and  $X2$ , the radii of the ellipse, are:

$$X1 = \sinh \left[ \frac{1}{N} \sinh^{-1} \left( \frac{1}{\epsilon} \right) \right] \quad (4a)$$

$$X2 = \cosh \left[ \frac{1}{N} \sinh^{-1} \left( \frac{1}{\epsilon} \right) \right]. \quad (4b)$$

Since it is inconvenient to work with hyperbolic functions, the standard mathematical relations<sup>13</sup>

$$\sinh x = (e^x - e^{-x}) / 2 \quad (5a)$$

$$\cosh x = (e^x + e^{-x}) / 2 \quad (5b)$$

$$\sinh^{-1} x = \ln [x + (x^2 + 1)^{1/2}] \quad (5c)$$

will be used to obtain the more convenient form:

$$X1 = [D - (1/D)] / 2 \quad (6a)$$

$$X2 = [D + (1/D)] / 2, \quad (6b)$$

where

$$D = \left[ \frac{1}{\epsilon} \left( \frac{1}{\epsilon^2} + 1 \right)^{1/2} \right]^{1/N} \quad (7)$$

The real and imaginary components of the pole locations for both types of filters can therefore be expressed in terms of ONE set of equations:

$$\sigma_i = X1 \cos(P_i) \quad (8a)$$

$$j\omega_i = jX2 \sin(P_i) \quad (8b)$$

where  $P_i$  is defined in Eq. 1.

If the number of poles is even, an  $N$ -pole function may be factored into  $N/2$  conjugate pairs, since each pole,  $s_i$ , has a conjugate pole  $s_{N-1-i}$ . Let

$$A_i \triangleq (\sigma_i + j\omega_i) (\sigma_i - j\omega_i) = \sigma_i^2 + \omega_i^2. \quad (9)$$

The normalized transfer function  $L_i(s)$  of a low-pass pole pair is then:

$$L_i(s) = \frac{1}{A_i} \frac{1}{\frac{1}{A_i} s^2 - \frac{2\sigma_i}{A_i} s + 1} \quad (10)$$

standard.

The second page of the print-out (Fig. 3B) is a plot of the logarithmic gain in decibels versus linear frequency in kilohertz. Plus signs are used to form the graph grid, and asterisks indicate the data points. The lines connecting the asterisks were drawn in after the print-out was completed to make the response curve more legible. The curve in Fig. 3B agrees with the standard six-pole Butterworth characteristic.<sup>9</sup>

The complete circuit schematic for the six-pole low-pass filter is shown in Fig. 3C. Three basic low-pass circuits have been cascaded (Fig. 1), with the component values of each section determined from Fig. 3A. A laboratory model of the circuit was built using the last active element

shown in Table 1. Tests showed that it did have the frequency response predicted in Fig. 3B.

In the example of the four-pole Chebyshev high-pass filter, line 0 in the computer program (Fig. 2) is in the correct format for solving the filter design. The solution is shown in Fig. 4. Figure 4A gives the component values, Fig. 4B the frequency response, and Fig. 4C the complete circuit schematic.

The headings provided in the computer print-out (Figs. 4A and 4B) for the Chebyshev filter are similar to those for the Butterworth case, but they include the definition of an additional parameter—the passband ripple. The plot of Fig. 4B shows every data point (rather than every other point as in Fig. 3B), to more clearly define

where the constant factor  $1/A_i$  in Eq. 10 normalizes the maximum value of  $L_i(s)$  to unity gain. Since the low-pass section (Fig. 1) must have unity gain at dc, the factor  $1/A_i$  cannot be accommodated by Eq. 12 for the Chebyshev case. Chebyshev filters designed in this way will have unity gain ripple minimums rather than the more

restriction also holds for high-pass design.

The normalized transfer function  $H_i(s)$  of a high-pass pole pair is obtained by replacing the variable  $s$  in (10) by  $1/s$ :

$$H_i(s) = \frac{1}{\frac{1}{A_i} \left( \frac{1}{A_i s^2} - \frac{2\sigma_i}{A_i s} + 1 \right)} \quad (11)$$

The transfer function  $L'(s)$  of the two-pole low-pass RC filter section (Fig. 1) is

$$L'(s) = \frac{1}{R_1 C_1 R_2 C_2 s^2 + (R_1 + R_2) C_2 s + 1} \quad (12)$$

To realize  $L_i(s)$  with  $L'(s)$ , Eqs. 10 and 12 must be equated term by term. If the constant factor is neglected, this leads to two equations in four unknowns,  $R_1$ ,  $R_2$ ,  $C_1$  and  $C_2$ :

$$R_1 C_1 R_2 C_2 = 1/A_i \quad (13a)$$

$$(R_1 + R_2) C_2 = -2 \sigma_i / A_i \quad (13b)$$

If  $C_1$  and  $C_2$  are selected as standard values,

$$C_1 = C \quad (14a)$$

$$C_2 = C/M, M > 1 \text{ for realizability,} \quad (14b)$$

then the normalized values of  $R_1$  and  $R_2$  are

determined from

$$R_1 = \frac{-\sigma_i M}{A_i C} \left[ 1 + \left( 1 - \frac{A_i}{\sigma_i^2 M} \right)^{1/2} \right] \quad (15a)$$

$$R_2 = \frac{-\sigma_i M}{A_i C} \left[ 1 - \left( 1 - \frac{A_i}{\sigma_i^2 M} \right)^{1/2} \right] \quad (15b)$$

From Eq. 15 it can be seen that the value of  $M$  selected in Eq. 14b must satisfy the inequality

$$M \geq A_i / \sigma_i^2 \quad (16)$$

The standard value of  $C_2$  may be found by first choosing  $C_2 > C$  and then trying successively smaller standard values for  $C_2$  until Eq. 16 is satisfied.

The transfer function  $H'(s)$  of the two-pole high-pass RC filter (Fig. 1) section is

$$H'(s) = \frac{1}{\frac{1}{R_1 C_1 R_2 C_2 s^2} + \frac{C_1 + C_2}{C_1 R_2 C_2 s} + 1} \quad (17)$$

As in the low-pass case,  $H_i(s)$  may be realized with  $H'(s)$  by selecting standard  $C$  values and then computing the normalized  $R$  values. The latter are derived by equating the denominators of Eqs. 11 and 17:

$$C_1 = C \quad (18a)$$

$$C_2 = C \quad (18b)$$

$$R_1 = \frac{-\sigma_i}{C} \quad (19a)$$

$$R_2 = \frac{-A_i}{\sigma_i C} \quad (19b)$$

The component value formulas are summarized in Table 2.

the faster-moving Chebyshev response.

The standard decade list of  $C$  values is not utilized in the high-pass design (Fig. 4A) since both  $C_1$  and  $C_2$  are set equal to the maximum  $C$  value specified in the DATA line. The high-pass graph (Fig. 4B) uses an inverse-linear frequency scale to emphasize the inverse symmetry of the high-pass design with respect to its low-pass prototype. The two filter sections of the complete schematic (Fig. 4C) use the basic high-pass rather than the low-pass circuit. ■■

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INSERTION GAIN (DB)

-15	-12	-9	-6	-3	0	3	
+ - - - - +	+ - - - - +	+ - - - - +	+ - - - - +	+ - - - - +	+ - - - - +	+ - - - - +	
+ + + + +	+ + + + +	+ + + + +	+ + + + +	+ + + + +	* + + + +	+ + + + +	= 0.040
					*		= 0.080
					*		= 0.120
					*		= 0.160
					*		= 0.200
					*		
+ + + + +	+ + + + +	+ + + + +	+ + + + +	+ + + + +	* + + + +	+ + + + +	= 0.280
					*		= 0.320
					*		= 0.360
					*		= 0.400
					*		
					*		= 0.480
+ + + + +	+ + + + +	+ + + + +	+ + + + +	+ + + + +	* + + + +	+ + + + +	= 0.520
					*		= 0.560
					*		= 0.600
					*		= 0.640
					*		= 0.680
					*		= 0.720
+ + + + +	+ + + + +	+ + + + +	+ + + + +	+ + + + +	* + + + +	+ + + + +	= 0.760
					*		= 0.800
					*		= 0.840
					*		= 0.880
					*		= 0.920
					*		= 0.960
+ + + + +	+ + + + +	+ + + + +	+ + + + +	+ + + + +	+* + + + +	+ + + + +	= 1.000
					*		= 1.040
					*		= 1.080
					*		= 1.120
					*		= 1.160
					*		= 1.200
+ + + + +	+ + + + +	+ + + + +	+ + + + +	+ + + + +	+ + + + +	+ + + + +	= 1.240
					*		= 1.280
					*		= 1.320
					*		= 1.360
					*		= 1.400
					*		= 1.440
+ + + + +	+ + + + +	+ + + + +	+ + + + +	+ + + + +	+ + + + +	+ + + + +	= 1.480
*							
*E							
*G							

T :H N :4 H :1 R :2.4 CI :10

C2 :10 R1 :1.527 R2 :151.98

C2 :10 R1 :3.68 R2 :14.38

INSERTION GAIN (DB)

=-15	=-12	=-9	=-6	=-3	= 0	= 3	
+ - - - +	+ - - - +	+ - - - +	+ - - - +	+ - - - +	+ - - - +	+ - - - +	
+	+	+	+	+	+	+	= 0.676
							= 0.695
							= 0.714
							= 0.735
							= 0.758
							= 0.781
+	*	+	+	+	+	+	= 0.807
		*					= 0.833
			*				= 0.862
				*			= 0.893
					*		= 0.926
						*	= 0.962
+	+	+	+	+	*	+	= 1.000
						*	= 1.042
						*	= 1.087
						*	= 1.136
						*	= 1.191
					*		= 1.250
+	+	+	+	+	*	+	= 1.316
					*		= 1.389
					*		= 1.471
					*		= 1.563
					*		= 1.667
					*		= 1.786
+	+	+	+	+	+	*	= 1.923
						*	= 2.083
						*	= 2.273
						*	= 2.500
						*	= 2.778
						*	= 3.125
+	+	+	+	+	+	*	= 3.572
						*	= 4.167
						*	= 5.000
						*	= 6.250
						*	= 8.333
						*	= 12.50
+	+	+	+	+	*	+	= 25.00
*							

