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DECUS NO.	FOCAL8-226
TITLE	FREQUENCY TRANSFORMATION PROGRAM
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SOURCE LANGUAGE	FOCAL 1969; PAL III

FREQUENCY TRANSFORMATION PROGRAM

DECUS Program Library Write-up

DECUS NO. FOCAL8-226

1. ABSTRACT:

Various Fourier transformation methods can be applied when using the Frequency Transformation Program described below. This program should examine in particular the accuracy of the Fast Fourier Transformation FOCAL program developed by ROTHMAN/2/ in comparison with normal Fourier transformations.

The result is that the Fast Fourier Transformation should be used, if the number of discrete points is $N = 2^{NU}$ (NU = 1,2,3,...). If not so, the transformation method with trapezoidal integration and lag window "hanning" should be used.

2. REQUIREMENTS:

2.1 Hardware: An 8-k PDP-8/I or 8/E computer with an ASR-33 teletype is the minimum hardware.

2.2 Software:

1) FOCAL 1969, DEC-08-AJAE-PB
   initial dialogue: NO - YES
2) Utility overlays for FOCAL 1969 (8-k)
   DEC-08-AJ1E-PB
3) if available: MODV-Choice,
   DECUS No. FOCAL 8-135
4) FNEW-Function for the Fast Fourier
   Transformation

3. LOADING PROCEDURE:

1) Load FOCAL 1969 with the BIN-Loader into field Ø and start FOCAL at location Ø2ØØ.
2) Answer the initial dialogue with NO - YES

3) Stop the computer. Load the 8-k overlay with the BIN-Loader into field 1.

4) If the program MODV-Choice is not available, leave out this point.

   Load the DECUS program MODV-Choice with the BIN-Loader into field Ø. Restart FOCAL at location Ø2ØØ and answer the question with Y or N. Stop the computer.

5) Load the FNEW-Function with the BIN-Loader into field Ø.

6) Restart FOCAL at location Ø2ØØ.

7) Load the FOCAL Frequency Transformation Program.

8) Start the FOCAL program with the GO command.

   The teletype will give a message and the program will erase the group 1 commands.

9) You have to write the lines 2.14 and 2.16 specially for your problem (for details see chapter 8: comment of the Frequency Transformation Program in the listings, group I of FOCAL program).

10) With a GO command you will start the transformation.

4. THEORY

4.1 Integration Methods

A certain integral

$$Y = \int_a^b y(t)\, dt$$

can be evaluated numerically only by approximation. In the different existing integration methods the formalism increases to some extent for more accuracy.

4.1.1 Trapez Integration

When evaluating an integral of a curve with only two ordinates $(t_o, y_o); (t_1, y_1)$, you get the greatest error. This linear interpolation (or trapezoidal integration)

$$y = \int_{t_0}^{t_1} y(t)\, dt \approx \frac{\Delta t}{2} (y_0 + y_1) \qquad \Delta t = t_1 - t_0$$

can be easily developed.

4.1.2 Simpson Integration

In the Simpson integration

$$y = \int_{t_0}^{t_0+2\cdot\Delta t} y(t)\, dt \approx \frac{2\Delta t}{6} [y_0 + 4y_1 + y_2]$$

an integral is evaluated with three ordinates (t_o, y_o) ; (t_1, y_1) ; (t_2, y_2) . If the integral has to be determined for a longer interval, the respective formulas can be expressed as follows (equation 1):

$$y = \int_{t_0}^{t_N} y(t)\, dt \approx \frac{\Delta t}{3} [y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots$$
$$\dots + 2y_{N-2} + 4y_{N-1} + y_N]$$

Here the interval is divided into a linear number N of segments of equal width Δt .

4.2 Fourier Transformation

With the Fourier transformation a function of the time domain is transformed into one of the frequency domain. If this function is only given in discrete ordinates, the frequency domain after the transformation is limited.

The frequency interval is:

$$\Delta f = 1/(2\cdot\Delta t\cdot N)$$

Here Δt is the time interval and N the number of ordinates. So the limited frequency domain after the transformation is

$$f_k = k \cdot \Delta f \qquad \text{for} \qquad k = 0, 1, \ldots, N$$

or

$$f_k = 0, \ \Delta f, \ 2 \cdot \Delta f, \ 3 \cdot \Delta f, \ \ldots, \ N \cdot \Delta f = 1/(2 \cdot \Delta t)$$

4.21 Fourier Integral Transformation

The equation

$$P(f) = \int_{-\infty}^{\infty} y(t)\, e^{-j \cdot \omega \cdot t}\, dt$$

shows the Fourier transform of a function y(t) .
If the function is non-existent for times $t < 0$, P(f) is as follows (equation 2):

$$P(f) = 2 \cdot \int_{0}^{\infty} y(t)\, e^{-j \cdot \omega \cdot t}\, dt$$

or it is divided into real and imaginary parts (equation3):

$$Re\ P(f) = 2 \cdot \int_{0}^{\infty} y(t) \cos(\omega t)\, dt$$

$$Im\ P(f) = -2 \int_{0}^{\infty} y(t) \sin(\omega \cdot t)\, dt$$

4.2.2 Derivation from a Fourier Series

Periodic variables can be represented in the time domain by a Fourier series of the form

$$y(t) = \frac{a_0}{N \cdot \Delta t} + \frac{2}{N \cdot \Delta t} \cdot \sum_{K=1}^{\infty} \left[a_K \cdot \cos(k \cdot \omega_1 \cdot t) + b_K \cdot \sin(k \cdot \omega_1 \cdot t) \right]$$

where the coefficients a_k and b_k are defined by (equation 4):

$$a_k = \int_0^{N \cdot \Delta t} y(t) \cos(\omega_k \cdot t)\, dt$$

$$b_k = \int_0^{N \cdot \Delta t} y(t) \sin(\omega_k \cdot t)\, dt$$

$N \cdot \Delta t$ is the periodic of the variable, and ω_k, the k-th harmonic of the fundamental frequency ω_1, is given by (equation 5):

$$\omega_k = k \cdot \omega_1 = 2 \cdot \pi \cdot k \cdot f_1 = \frac{2 \cdot \pi \cdot k}{N \cdot \Delta t}$$

If the variable $y(t)$ is sampled at N equally spaced points Δt seconds apart, the Fourier transform is /3/ (equation 6):

$$Re\ P(f) = \frac{\Delta t}{2}\left[y(0) + 2 \cdot \sum_{i=1}^{N-1} y(t_i) \cos \frac{\pi i k}{N} + (-1)^k y(t_N)\right]$$

$$Im\ P(f) = -\Delta t \sum_{i=1}^{N-1} y(t_i) \sin \frac{\pi i k}{N} \qquad k = 0, 1, 2, \ldots, N$$

Here the coefficients a_k and b_k are evaluated by trapezoidal integration.

4.2.3 Fast Fourier Transformation

The Fast Fourier Transformation is extremely useful in the convolution of time series. The algorithm has been well described by BRIGHAM /1/ and ROTHMAN /2/. Since the Fast Fourier Transformation FOCAL program developed by ROTHMAN /2/ is used in this Frequency

Transformation Program too, there will be no detailed description of the Fast Fourier Transformation here.

4.3 Lag Windows

When using the Fourier transformation from a number of discrete ordinates, there are usually side lobes apart from the main lobe. In order to concentrate the main lobe and keep the side lobe as low as possible, the ordinates of the function can be multiplied by a lag window.

The lag window (equation 7)

$$D_o(t) = 1 \qquad \text{for } |t| \leq t_{max}$$

$$= 0 \qquad \text{for } |t| > t_{max}$$

is practically of no consequence. A simple and convenient compromise is represented by the lag window called "hanning" (equation 8):

$$D_2(t) = 0.5\left(1 + \cos\frac{\pi \cdot t}{t_{max}}\right) \qquad \text{for } |t| \leq t_{max}$$

$$= 0 \qquad \text{for } |t| > t_{max}$$

An alternative compromise is represented by the lag window "hamming" (equation 9):

$$D_3(t) = 0.54 + 0.46\cos\frac{\pi \cdot t}{t_{max}} \qquad \text{for } |t| \leq t_{max}$$

$$= 0 \qquad \text{for } |t| > t_{max}$$

5. Frequency Transformation Program

The present FOCAL program transforms a number of ordinates (time domain) into the frequency domain. Here different integration and transformation methods are used.

1) Simpson integration:

The Fourier transformation takes place according to equation 3. Here the integration method is the Simpson integration (equation 1).

2) Simpson integration with "hanning" window:

In addition to the above method 1 the lag window $D_2(t)$ "hanning" (equation 8) is considered to smooth the curve.

3) Simpson integration with "hamming" window:

In addition to method 1 the lag window $D_3(t)$ "hamming" (equation 9) is considered.

4) Trapez integration:

The Fourier transformation takes place according to equation 6. Here the integration method is the trapezoidal integration.

5) Trapez integration with "hanning" window:

If the lag window $D_2(t)$ "hanning" is considered in equation 6, it is reduced to (equation 10):

$$Re\ P(f) = \frac{\Delta t}{2}\left[y(0) + \sum_{i=1}^{N-1}\left(1+\cos\frac{\pi i}{N}\right) y(t_i) \cos\frac{\pi i k}{N}\right]$$

$$Im\ P(f) = -\frac{\Delta t}{2}\sum_{i=1}^{N-1}\left(1+\cos\frac{\pi i}{N}\right) y(t_i) \sin\frac{\pi i k}{N}$$

$$k = 0, 1, 2, \ldots, N$$

6) Fast Fourier Transformation:

The Fast Fourier Transformation according to ROTHMAN /2/ is made.

6. Comparison of the Different Methods

The quality of a Fourier transformation method was seen in the errors that occur when a theoretical peak (the amplitude of the peak >>> the amplitude of the harmonics; width of the peak approximately zero) is evaluated by different methods.

With the Frequency Transformation Program several operations were made to find out the quality of the individual methods, particularly that of the Fast Fourier Transformation. The result was as follows (the percentages below refer to the example described in chapter 8):

1) The Simpson integration has better results (as could have been expected from the theory) than the trapezoidal integration (without using a lag window).

   The Simpson integration has a sharp peak at $\omega_1 = 1$ (1/sec) however the ratio of the amplitudes (the amplitudes of the higher harmonics to the amplitude of the main lobe) is appr. 8 per cent. There is an additional peak at $6,5 \cdot \omega_1$ (particularly in this example).

   The trapezoidal integration has a somewhat wider (and thus less correct) main lobe; the ratio of the amplitudes is appr. 10 per cent; however there is no additional peak.

2) When using lag windows in the Fourier transformation, the curve is smoothed, i. e. the amplitudes of the higher harmonics are reduced, whereas the main lobe becomes wider.

In the Simpson integration the lag windows "hanning" and "hamming" are used. The "hanning" window (at the ratio of the amplitudes of appr. zero) widens the main lobe a bit more than the "hamming" window (at the ratio of the amplitudes of appr. 1,5 per cent). The additional peak at $6,5 \cdot \omega_1$ was again there.

3) If the lag window "hanning" is used in the trapezoidal integration, the main lobe is also wider, however the ratio of the amplitudes is only about 0.2 per cent. There is no additional peak. Apart from that, the imaginary part (theoretically equal zero in the example) is much smaller than in the Simpson integration.

4) The Fast Fourier Transformation shows a sharp main lobe; the amplitudes of the higher harmonics are reduced very quickly whereas the frequency ω_k increases. The imaginary part is evaluated better than with the above mentioned methods.

The comparison of the various methods shows that the Fast Fourier Transformation FOCAL program developed by ROTHMAN /2/ has not only a much higher operating speed, but that it is also more accurate than the other described methods. So it should be used, if the Fourier transform is to be evaluated out of a time series of $N = 2^{NU}$ discrete ordinates. However it should be kept in mind, that the frequency step is

$$\Delta f = 1/(N \cdot \Delta t)$$

in the Fast Fourier Transformation FOCAL program and that the results for the frequencies

$$f > 1/(2 \cdot \Delta t)$$

cannot be regarded as exact.

However, if there is a number of $N \neq 2^{NU}$ ordinates, the trapez integration with lag window $D_2(t)$ "hanning", (equation 10), is useful, since this method is comparatively exact. Here the frequency step is

$$\Delta f = 1/(2 \cdot \Delta t \cdot N)$$

and the maximum frequency:

$$f_{max} = 1/(2 \cdot \Delta t)$$

7. LITERATURE

/1/ BRIGHAM, E.O.; MORROW, R.E.:
The fast Fourier transform
IEEE Epectrum, Dec. 1967, p. 63 - 70

/2/ ROTHMAN, J. E.:
The Fast Fourier Transform and its Implementation
DECUSCOPE 1968, Vol. 7, No. 3, p. 3 - 10

/3/ UHRIG, R.E.: Random Noise Techniques in Nuclear Reactor Systems
Ranold Press Comp., New York

8. LISTINGS OF PROGRAMS

The Fast Fourier Transformation FOCAL program written by ROTHMAN /2/ will be repeated once more below, because there are different, sometimes even incorrect versions in the various existing publications.

The following listings are attached:

1) Listing of the FNEW-Function for the Fast Fourier Transformation
2) Listing of the FOCAL Frequency Transformation Program
3) The teletype output when starting the Frequency Transformation Program and an example.

```
/ PROGRAM: SUBROUTINE FNEW FOR FOCAL 1969, VERSION AJAE
/                    INITIAL DIALOGUE: NO-YES

/                    FNEW(X,Y)   FOR FAST FOURIER TRANSFORMATION


/ FOCAL SPECIFICATION

EFUN3I=135
EVAL=1613
FLAC=44
GETC=4545
INTEGER=53
PUSHJ=4540
SPNOR=4560



            *35
0035  5152  BOTTOM,   XFNEW-1

            *410
0410  5153            XFNEW              / FOCAL VERSION AJAE

            *5153
5153  4453  XFNEW,    JMS I INTEGER      / FIX FLOATING POINT AC
5154  1045            TAD FLAC+1         / TAKE FIRST ARGUMENT
5155  7041            CIA
5156  3377            DCA CNTR           / NUMBER OF BITS
5157  4560            SPNOR              / MOVE PAST SPACES
5160  4545            GETC               / GET PAST COMMA
5161  4540            PUSHJ              / EVALUATE NEXT ARGUMENT
5162  1613            EVAL
5163  4453            JMS I INTEGER      / FIX FL.P.AC
5164  3045            DCA FLAC+1         / PUT IT IN FLAC+1
5165  3046            DCA FLAC+2         / BUILD UP RESULT IN FLAC+2
5166  1045  LOOP,     TAD FLAC+1         / TRANSPOSE 45 ABOUT ITS CENTB
5167  7110            CLL RAR            / BY ROTATING 45 RIGHT
5170  3045            DCA FLAC+1         /       / AND 46 LEFT
5171  1046            TAD FLAC+2         / BIT TO BE TRANSPOSE IN LINK
5172  7004            RAL                / INSERT INTO LOW ORDER BIT
5173  3046            DCA FLAC+2         /       / OF 46
5174  2377            ISZ CNTR           / FOR ALL NU BITS
5175  5366            JMP LOOP
5176  5536            JMP I EFUN3I       / RETURN TO MAIN PROGRAM

5177  0000  CNTR,     0

            *1217
1217  7200            7200               / NO ":" BY ASK COMMAND

            *6002
6002  7200            7200               / NO "=" BY TYPE COMMAND
```

```
01.01 C          FREQUENCY TRANSFORMATION PROGRAM
01.02 C
01.03 C
01.04 C
01.05 C LANGUAGE: FOCAL 1969, INITIAL DIALOGUE NO-YES
01.06 C           8-K FOCAL AND (NOT NECESSARY)
01.07 C           MODV-CHOICE (DECUS NO. 8-135)
01.08 C
01.09 C
01.10 C FUNCTION: FNEW(X,Y) FOR THE FAST FOURIER TRANSFORMATION
01.11 C
01.12 C A NUMBER OF DATA (TIME DOMAIN) ARE TRANSFORMED INTO A
01.14 C FREQUENCY DOMAIN. SIX DIFFERENT CALCULATIONS WILL TRANSFORM
01.16 C THE DATA:
01.18 C         1.) SIMPSON-INTEGRATION
01.20 C         2.) SIMPSON-INTEGRATION AND HANNING-WINDOW
01.22 C         3.) SIMPSON-INTEGRATION AND HAMMING-WINDOW
01.24 C         4.) TRAPEZ-INTEGRATION
01.26 C         5.) TRAPEZ-INTEGRATION AND HANNING-WINDOW
01.28 C         6.) FAST FOURIER TRANSFORMATION
01.29 C
01.30 C
01.31 C
01.32 C DATA INPUT: 1.) INPUT OF THE NUMBER OF DATA-ARRAYS
01.34 C             2.) DATA-ARRAY INPUT FROM HIGH SPEED READER
01.36 C OUTPUT OF DATA: THE TIME, DATA, FREQUENCY, REAL AND
01.38 C                 IMAGINARE PART OF THE FOURIERKOEFFICIENTS
01.39 C
01.40 C
01.41 C
01.42 C YOU HAVE TO WRITE NEW LINES  NO. 2.14  AND  NO. 2.16
01.44 C IF THE TIME INTERVAL IS  0.0666  SECONDS AND THE NUMBER
01.46 C OF DATA  N  [N(MAX)=128] IS PUNCHED ON PAPER TAPE,
01.47 C THEN WRITE:
01.48 C  02.14 S T1=0.0666
01.50 C  02.16 *; A N;*
01.51 C
01.52 C
01.53 C
01.54 T !!!!,"PLEASE WRITE THE NEW LINES",!,"          "
01.56 T "NO. 02.14",!,"          NO. 02.16",!,"SPECIALLY FOR YOUR "
01.58 T "PROBLEM !",!!,"YOU HAVE TO DEFINE THE VARIABLES:",!
01.60 T "T1       TIME INTERVAL",!,"N        NUMBER OF DATA",!!
01.61 C
01.62 C
01.63 C
01.64 E 1
```

```
Ø2.Ø1 C FREQUENCY TRANSFORMATION PROGRAM
Ø2.Ø2 C
Ø2.Ø3 T !!!!!
Ø2.1Ø A "NUMBER OF DATA-ARRAYS ? ",KU; I [KU] 2.99,2.99,2.12
Ø2.12 T !!!!!!!!!!,"DATA-ARRAY:",%2.ØØ,KU,!!!!!
Ø2.14 T "PLEASE NEW LINES !!!",!!,"Ø2.14  ???",!
Ø2.16 T "Ø2.16  ???",!!; Q
Ø2.18 S TM=(N-1)*T1
Ø2.2Ø S DF=1/TM/2; S P1=3.141593; S P2=6.283186
Ø2.22 T "NUMBER OF DATA =",%3.ØØ,N,!
Ø2.24 T "DELTA-T   =",%7.Ø6,T1,"  [SEC]",!
Ø2.25 T "TAU(MAX)  =",%7.Ø3,TM,"  [SEC]",!
Ø2.26 T "DELTA-F   =",%7.Ø6,DF,"  [HZ]",!!!!!
Ø2.28 *; F I=Ø,N-1; A A(I)
Ø2.3Ø *
Ø2.32 T "DATA-ARRAY",!!,"    T [SEC]         DATA",!!
Ø2.34 F I=Ø,N-1; T %7.Ø3,I*T1,"      ",%7.Ø6,A(I),!
Ø2.36 T !!!!
Ø2.5Ø C
Ø2.52 T "SIMPSON-INTEGRATION"; D 8
Ø2.54 S D1=-1; S D2=1.; S D3=1.; D 3
Ø2.56 C
Ø2.58 T "SIMPSON-INTEGRATION    HANNING"; D 8
Ø2.6Ø S D1=Ø; S D4=Ø.5; S D5=Ø.5; D 3
Ø2.62 C
Ø2.64 T "SIMPSON-INTEGRATION    HAMMING"; D 8
Ø2.66 S D4=Ø.54; S D5=Ø.46; D 3
Ø2.68 C
Ø2.7Ø T "TRAPEZ-INTEGRATION"
Ø2.72 D 8; D 4
Ø2.74 C
Ø2.76 T "TRAPEZ-INTEGRATION      HANNING"
Ø2.78 D 8; D 5
Ø2.8Ø C
Ø2.82 T "FAST FOURIER TRANSFORMATION"
Ø2.84 S NU=Ø; I [N-128] 2.86,2.86; S N=128
Ø2.86 S NU=NU+1; I [(2↑NU)-N] 2.88,2.9Ø; S NU=NU-1; G 2.9Ø
Ø2.88 G 2.86
Ø2.9Ø I [NU] 2.99,2.99; S N=2↑NU; D 16
Ø2.99 S KU=KU-1; I [-KU] 2.12; T !!!!!!!!!!; Q

Ø3.Ø1 C SIMPSON INTEGRATION
Ø3.Ø2 C
Ø3.1Ø S K=Ø
Ø3.12 S OM=K*DF; S T=T1; S H=OM*P2*T
Ø3.14 I [D1] 3.18; D 1Ø
Ø3.18 S RP=A(Ø)+4*D2*A(1)*FCOS(H)
Ø3.2Ø S IP=4*D2*A(1)*FSIN(H)
Ø3.22 S T=T+T1; F I=2,2,N-2; D 11
Ø3.26 S RP=2*T1*RP/3; S IP=-2*T1*IP/3; D 12
Ø3.28 S K=K+1; I [K-N] 3.12,3.12; T !!!!!; R
```

```
04.01 C TRAPEZ-INTEGRATION
04.02 C
04.10 S K=0
04.12 D 13
04.14 F I=1,N-2; D 14
04.18 S RP=(T1/2)*[A(0)+2*RP+A(N-1)*((-1)↑K)]
04.20 S IP=-(T1)*IP; D 12
04.22 S K=K+1; I [K-N] 4.12,4.12; T !!!!!; R

05.01 C TRAPEZ-INTEGRATION        HANNING
05.02 C
05.10 S K=0
05.12 D 13
05.14 F I=1,N-2; D 15
05.18 S RP=(T1/2)*[A(0)+RP]
05.20 S IP=-(T1/2)*IP; D 12
05.22 S K=K+1; I [K-N] 5.12,5.12; T !!!!!; R

08.01 C OUTPUT
08.02 C
08.10 T !!,"   F [HZ]             REAL                          IMAGINARE",!
08.12 T "                      PART                          PART",!!

10.01 C HANNING    -    HAMMING
10.02 C
10.10 S D2=D4+D5*FCOS[P1*T/TM]
10.12 S D3=D4+D5*FCOS[P1*(T+T1)/TM]

11.01 C FREQUENCY TRANSFORMATION
11.02 C SIMPSON INTEGRATION
11.03 C
11.10 S H=OM*P2*T; S H1=OM*P2*(T+T1)
11.12 I [D1] 11.16; D 10
11.16 S RP=RP+2*D2*A(I)*FCOS(H)+4*D3*A(I+1)*FCOS(H1)
11.18 S IP=IP+2*D2*A(I)*FSIN(H)+4*D3*A(I+1)*FSIN(H1)
11.20 S T=T+T1+T1

12.01 C OUTPUT ON TELETYPE
12.02 C
12.12 T %7.04,OM,"      ",%,RP,"       ",IP,!

13.01 C PARAMETER
13.02 C
13.10 S RP=0; S IP=0
13.12 S OM=(*)F

14.01 C FREQUENCY TRANSFORMATION
14.02 C TRAPEZ-INTEGRATION
14.03 C
14.10 S H=P1*I*K/N
14.14 S RP=RP+A(I)*FCOS(H)
14.16 S IP=IP+A(I)*FSIN(H)

15.01 C FREQUENCY TRANSFORMATION
15.02 C TRAPEZ-INTEGRATION        HANNING
15.03 C
15.10 S H=P1*I*K/N; S H1=1+FCOS(P1*I/N)
15.14 S RP=RP+A(I)*H1*FCOS(H)
15.16 S IP=IP+A(I)*H1*FSIN(H)
```

```
16.01 C FAST FOURIER TRANSFORMATION
16.02 C
16.10 S T=P2/N;  S S=N/2;  S L=1;  S Q=S-1;  S H=1-NU
16.12 F I=0,N-1;  S X(I)=0.
16.14 S SR=A(Q+S)+A(Q);  S A(Q+S)=A(Q)-A(Q+S);  S A(Q)=SR
16.16 I [Q] 16.18,16.18;  S Q=Q-1;  G 16.14
16.18 I [L-NU] 16.20,16.42,16.20
16.20 S L=L+1;  S S=S/2;  S H=H+1;  S P=N-1;  S Z=1/[2↑(-H)]
16.22 S C=1
16.24 S U=FITR(P*Z);  S K=T*FNEW(NU,U)
16.26 S CO=FCOS(K);  S SN=FSIN(K)
16.28 S GR=CO*A(P)+SN*X(P);  S GI=CO*X(P)-SN*A(P)
16.30 S Q=P-S;  S SR=GR+A(Q);  S SI=GI+X(Q);  S A(Q)=A(Q)-GR
16.32 S X(Q)=X(Q)-GI;  S A(P)=SR;  S X(P)=SI
16.34 S P=P-1;  I [C-S] 16.36,16.38,16.36
16.36 S C=C+1;  G 16.24
16.38 I [P-S+1] 16.40,16.18,16.40
16.40 S P=P-S;  G 16.22
16.42 D 8
16.50 S DF=1/(T1*(N-1));  F I=0,N-1;  D 17

17.01 C OUTPUT OF FFT
17.02 C
17.10 T %7.04,I*DF,"        ";  S K=FNEW(NU,I)
17.12 S SR=2*A(K)/N;  S SI=2*X(K)/N
17.14 T %,SR,"          ",SI,!
*




GO




PLEASE WRITE THE NEW LINES
        NO. 02.14
        NO. 02.16
SPECIALLY FOR YOUR PROBLEM !

YOU HAVE TO DEFINE THE VARIABLES:
T1        TIME INTERVAL
N         NUMBER OF DATA

*2.14 S T1=0.066667
*2.16 *;  A N;*
*GO
```

NUMBER OF DATA-ARRAYS ? 1

DATA-ARRAY: 1

NUMBER OF DATA = 16
DELTA-T = 0.066667 [SEC]
TAU(MAX) = 1.000 [SEC]
DELTA-F = 0.499993 [HZ]

DATA-ARRAY

T [SEC]	DATA
0.000	1.000000
0.067	0.913545
0.133	0.669127
0.200	0.309011
0.267	-0.104537
0.333	-0.500010
0.400	-0.809025
0.467	-0.978152
0.533	-0.978146
0.600	-0.809008
0.667	-0.499984
0.733	-0.104506
0.800	0.309039
0.867	0.669150
0.933	0.913557
1.000	1.000000

SIMPSON-INTEGRATION

F [HZ]	REAL PART	IMAGINARE PART
0.0000	0.888975E-01	0.000000E+00
0.5000	-0.889030E-01	0.429110E+00
1.0000	0.198890E+01	-0.987623E-02
1.5000	-0.888797E-01	-0.749774E+00
2.0000	0.888845E-01	-0.208675E-01
2.5000	-0.888862E-01	-0.279222E+00
3.0000	0.888870E-01	-0.345729E-01
3.5000	-0.888880E-01	-0.163909E+00
4.0000	0.888880E-01	-0.546335E-01
4.5000	-0.888884E-01	-0.103726E+00
5.0000	0.888881E-01	-0.930716E-01
5.5000	-0.888884E-01	-0.626099E-01
6.0000	0.888878E-01	-0.249917E+00
6.5000	-0.422227E+00	-0.296363E-01
7.0000	0.888911E-01	0.143044E+00
7.5000	-0.888925E-01	-0.460916E-05
8.0000	0.888945E-01	-0.143030E+00

SIMPSON-INTEGRATION HANNING

F [HZ]	REAL PART	IMAGINARE PART
0.0000	-0.283654E-05	0.000000E+00
0.5000	0.249998E+00	0.212836E+00
1.0000	0.500002E+00	-0.851041E-01
1.5000	0.250006E+00	-0.382572E+00
2.0000	0.179311E-06	-0.267683E+00
2.5000	-0.124838E-06	-0.153471E+00
3.0000	-0.464961E-06	-0.128070E+00
3.5000	0.131608E-06	-0.104256E+00
4.0000	-0.456186E-06	-0.942255E-01
4.5000	0.439629E-06	-0.887390E-01
5.0000	-0.537439E-06	-0.881195E-01
5.5000	0.189992E-06	-0.117052E+00
6.0000	-0.833349E-01	-0.148033E+00
6.5000	-0.166668E+00	-0.415616E-01
7.0000	-0.833343E-01	0.640992E-01
7.5000	0.726518E-06	0.181664E-05
8.0000	-0.833324E-01	-0.640991E-01

SIMPSON-INTEGRATION HAMMING

F [HZ]	REAL PART	IMAGINARE PART
0.0000	0.710911E-02	0.000000E+00
0.5000	0.222886E+00	0.229448E+00
1.0000	0.547114E+00	-0.790860E-01
1.5000	0.222895E+00	-0.411948E+00
2.0000	0.711093E-02	-0.247938E+00
2.5000	-0.711100E-02	-0.163532E+00
3.0000	0.711052E-02	-0.120590E+00
3.5000	-0.711092E-02	-0.109029E+00
4.0000	0.711066E-02	-0.910580E-01
4.5000	-0.711066E-02	-0.899839E-01
5.0000	0.711054E-02	-0.835157E-01
5.5000	-0.711089E-02	-0.112697E+00
6.0000	-0.695571E-01	-0.156184E+00
6.5000	-0.187112E+00	-0.406116E-01
7.0000	-0.695563E-01	0.704148E-01
7.5000	-0.711075E-02	0.130258E-05
8.0000	-0.695542E-01	-0.704136E-01

TRAPEZ-INTEGRATION

F [HZ]	REAL PART	IMAGINARE PART
0.0000	0.383061E-05	0.000000E+00
0.5000	-0.195143E-01	0.193071E+00
1.0000	0.474914E+00	0.312137E-01
1.5000	0.119744E+00	-0.394724E+00
2.0000	-0.709736E-01	-0.570135E-01
2.5000	0.495619E-01	-0.927210E-01
3.0000	-0.330182E-01	-0.666087E-01
3.5000	0.177993E-01	-0.216885E-01
4.0000	-0.315498E-02	-0.698222E-01
4.5000	-0.109137E-01	0.895736E-02
5.0000	0.241403E-01	-0.636467E-01
5.5000	-0.361665E-01	0.193313E-01
6.0000	0.466199E-01	-0.483982E-01
6.5000	-0.551533E-01	0.167305E-01
7.0000	0.614731E-01	-0.261123E-01
7.5000	-0.653569E-01	0.643646E-02
8.0000	0.666671E-01	0.768253E-06

TRAPEZ-INTEGRATION HANNING

F [HZ]	REAL PART	IMAGINARE PART
0.0000	-0.975528E-02	0.000000E+00
0.5000	0.108972E+00	0.119339E+00
1.0000	0.262514E+00	-0.355677E-02
1.5000	0.160857E+00	-0.191312E+00
2.0000	0.683958E-02	-0.150368E+00
2.5000	-0.121700E-02	-0.772660E-01
3.0000	0.331308E-03	-0.619068E-01
3.5000	-0.143569E-03	-0.449520E-01
4.0000	0.144053E-03	-0.380939E-01
4.5000	-0.210517E-03	-0.283336E-01
5.0000	0.300143E-03	-0.247512E-01
5.5000	-0.393273E-03	-0.183456E-01
6.0000	0.479998E-03	-0.151834E-01
6.5000	-0.553468E-03	-0.102624E-01
7.0000	0.609051E-03	-0.726439E-02
7.5000	-0.643495E-03	-0.330972E-02
8.0000	0.654883E-03	0.151119E-06

FAST FOURIER TRANSFORMATION

F [HZ]	REAL PART	IMAGINARE PART
0.0000	0.125009E+00	0.000000E+00
1.0000	0.100595E+01	0.200112E+00
2.0000	-0.446361E-01	-0.185110E-01
3.0000	-0.140756E-01	-0.940476E-02
4.0000	-0.591507E-02	-0.591556E-02
5.0000	-0.257384E-02	-0.335224E-02
6.0000	-0.976011E-03	-0.235310E-02
7.0000	-0.224233E-03	-0.112623E-02
8.0000	0.298023E-07	0.000000E+00
9.0000	-0.223875E-03	0.112477E-02
10.0000	-0.976071E-03	0.235319E-02
11.0000	-0.257321E-02	0.335117E-02
11.9999	-0.591503E-02	0.591556E-02
12.9999	-0.140743E-01	0.940389E-02
13.9999	-0.446862E-01	0.185109E-01
14.9999	0.100595E+01	-0.200109E+00