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DECUS NO.	FOCAL8-165
TITLE	F-(VARIANCE RATIO) DISTRIBUTION PROBABILITY
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SOURCE LANGUAGE	FOCAL

F-(VARIANCE RATIO) DISTRIBUTION PROBABILITY

DECUS Program Library Write-up

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By: Alan S. Fields

PURPOSE: To calculate the probability that the variance ratio, or F - statistic occurred by chance. The F statistic is

$$F = \frac{X_1^2/V_1}{X_2^2/V_2}$$

where X_1^2, X_2^2 are independent random variables, and

V_1, V_2 are the associated degrees of freedom.

USAGE: The program operates within any FOCAL(1) system with the extended functions FEXP and FLOG.

METHOD: The distribution of F follows the F- or variance ratio distribution. The distribution function is

$$P(F/V_1,V_2) = \frac{V_1^{\frac{1}{2}V_1} V_2^{\frac{1}{2}V_2}}{B(\frac{1}{2}V_1, \frac{1}{2}V_2)} \int_0^F t^{\frac{1}{2}(V_1-2)} (V_2+V_1 t)^{-\frac{1}{2}(V_1+V_2)} dt$$

where $F \geq 0$,

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$ (the Beta function), and

$\Gamma(\cdot)$ is the gamma function.

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The probability of interest is

$$Q(F/v_1,v_2) = 1 - P(F/v_1,v_2)$$

which is the probablity that F occurred by chance. This probability should be small in a successful experiment. This probability is

$$Q(F/v_1,v_2) = I_x\left(\frac{v_2}{2}, \frac{v_1}{2}\right)$$

where $x = \dfrac{v_2}{v_2 + v_1 F}$, and

$I_x(a,b)$ is the incomplete Beta Function.

The incomplete Beta Function is

$$I_x(a,b) = \frac{1}{B(a,b)} \int_0^x t^{a-1} (1-t)^{b-1} dt$$

$$= \frac{x^a (1-x)^b}{a\, B(a,b)} \left\{ 1 + \sum_{n=0}^{\infty} \frac{B(a+1, n+1)}{B(a+b, n+1)} x^{n+1} \right\}$$

where, in each case $0 \leq x \leq 1$ which follows from $F \geq 0$.

The term to be summed can be calculated from a simple recursion relation as shown below.

$$\frac{B(a+1,n+1)}{B(a+b,n+1)} = \frac{\Gamma(a+1)\,\Gamma(n+1)\,\Gamma(a+b+n+1)}{\Gamma(a+1+n+1)\,\Gamma(a+b)\,\Gamma(n+1)}$$

$$= \left[\frac{\Gamma(a+1)}{\Gamma(a+1+n+1)}\right]\left[\frac{\Gamma(a+b+n+1)}{\Gamma(a+b)}\right]$$

$$= \left[\prod_{\rho=0}^{n}\frac{1}{(a+1+\rho)}\right]\left[\prod_{\rho=0}^{n}(a+b+\rho)\right]$$

$$f_n = \frac{B(a+1+n+1)}{B(a+b+n+1)}\,x^{n+1} = x^{n+1}\prod_{\rho=3}^{n}\left(\frac{a+b+\rho}{a+1+\rho}\right)$$

$$f_{n+1} = x\left[\frac{a+b+n+1}{a+1+n+1}\right] f_n$$

The gamma function is characterized by the following:

$$\Gamma(x+1) = x\,\Gamma(x)\,;\quad \text{all } x$$

$$\Gamma(x+1) = 1 - .574864\,x + .951236\,3x^2 - .6998588\,x^3 + .4245549\,x^4 - .1010678\,x^5\,;\quad 0 \le x \le 1$$

The basic relationships used here can be found in a number of books including the " Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables" published by the U. S. Department of Commerce, National Bureau of Standards, Number 55 in the Applied Mathematics Series.

PROGRAM DETAILS:

The only part of the program that does not directly follow from the above is the determination of the number of terms to be summed. As, generally, $F > 0$ so that $X < 1$, each suceeding term in the sum on n is smaller than its predecessor. The summation is stopped by an "IF" statement when the contribution becomes .00001 or less.

F - DISTRIBUTION, PROGRAM LISTING

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C-FOCAL , 8/68

01.01 C F DISTRIBUTION ROUTINE
01.02 C A.S.FIELDS   US NSRDL-ANNAPOLIS  6/29/70
01.10 A !!! "F-RATIO ",RF,!," FIRST DEGREES OF FREEDOM ",D1,!
01.20 A "SECOND DEGREES OF FREEDOM ",D2,!
01.30 D 2; T %5.04,"PROBABILITY OF CHANCE ",P; G 1.1

02.01 C CONTROL
02.10 S A=D2/2; S B=D1/2
02.20 S X=A; D 3; S QF=1/R; S X=B; D 3; S QF=QF/R
02.30 S X=A+B; D 3; S QF=QF*R; S X=D2/[D2+RF*D1]
02.40 S QF=QF*FEXP[FLOG(X)*A]*FEXP[FLOG(1-X)*B]/A
02.50 S Q=QF; S QQ=1; S N=-1
02.60 S N=N+1; D 4; S QF=QF*X; S Q=Q+QF*QQ
02.70 I (QF*QQ-1E-5)2.8,2.6,2.6
02.80 S P=Q

03.01 C GAMMA FUNCTION
03.10 S R=1
03.20 I (X)3.99,3.99; I (X-1)3.3,3.4,3.4
03.30 S R=R/X; S X=X+1
03.40 I (X-2)3.6,3.6,3.5
03.50 S X=X-1; S R=R*X; G 3.4
03.60 S X=X-1; S XX=1-.57486*X+.95123*X↑2-.69986*X↑3
03.70 S XX=XX+.42455*X↑4-.10106*X↑5; S R=R*XX; R
03.99 Q

04.01 C RECURSIVE B(A,B)
04.10 S QQ=QQ*(A+B+N)/(A+1+N)
*
*
*C SOME TYPICAL RUNS
*
*E
*G



F-RATIO :.983
 FIRST DEGREES OF FREEDOM :12
SECOND DEGREES OF FREEDOM :17
PROBABILITY OF CHANCE = 0.5003


F-RATIO :1.89
 FIRST DEGREES OF FREEDOM :8
SECOND DEGREES OF FREEDOM :5
PROBABILITY OF CHANCE = 0.2504


F-RATIO :999.4
 FIRST DEGREES OF FREEDOM :15
SECOND DEGREES OF FREEDOM :2
PROBABILITY OF CHANCE = 0.0010
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