



DECUS

PROGRAM LIBRARY

DECUS NO.	FOCAL8-163
TITLE	ERLANG C BLOCKING PROBABILITY PROGRAMS
AUTHOR	Richard R. Plum
COMPANY	Traffic Systems Engineering Department Bell Telephone Laboratories, Inc. Holmdel, New Jersey
DATE	March 17, 1971
SOURCE LANGUAGE	FOCAL

ERLAND C BLOCKING PROBABILITY PROGRAMS

DECUS Program Library Write-up

DECUS NO. FOCAL8-163

Abstract

Three different "FOCAL" programs are offered which compute the following:

Program No. 1 - Computes the Erlang C blocking probability given the offered load in Erlangs and the number of servers.

Program No. 2 - Computes the Erlang C blocking probability, the average delay in seconds, and the average holding time in seconds given the offered load in Erlangs, the number of servers, and the total number of requests for service (calls).

Program No. 3 - Computes the Erlang C blocking probability, the average delay in seconds, the average holding time in seconds, the delay greater than 10 seconds and the percent delayed greater than 10 seconds given the offered load in Erlangs, the number of servers, and the total number of requests for service (calls).

Minimum Hardware Required

PDP8-4K

Programming Language

FOCAL

Loading

1. Load FOCAL - 1969 and retain EXP function.
2. Load Erlang C program desired and type "GO".

Using

1. Input the requested data via the TTY keyboard, i.e., the number of Erlangs, Servers, and Calls.

2. The resultant computations will be typed on the TTY by the program preceded by appropriate labels.

3. Type "GO" to restart the program.

Examples

1. Program input and output examples are attached as Figures 1 and 2.

Programs

1. The program writeups are attached as Figures 3, 4, and 5.

Description

The Erlang C blocking probability is computed from the following formula:

$$\bar{d} = C(c, a) \cdot \frac{H}{(c-a)}$$

The delay greater than 10 seconds is computed as follows:

$$P(D > 10) = C(c, a) e^{-(c-a) \frac{10}{H}}$$

c - Number of servers⁽¹⁾

a = Offered load⁽²⁾ in Erlangs⁽³⁾

C(c, a) = Erlang C blocking probability

H = Average holding time in seconds

(1) Servers = Number of trunks, operators, or such, to which the offered load is submitted.

(2) Offered Load = Number of requests for service (calls) times the average serving time in seconds.

(3) One Erlang = 1 call hour = 3600 call seconds.

The programs were designed primarily for Telephone Company Traffic Engineering use but they may be used in any serving

problem where the request for service, once initiated, stays in the system until served. Turnpike or tunnel toll booths and supermarket checkout counters are prime examples of the Erlang C assumption. In real situations, the relationship of average delay to percent greater than 10 seconds tends to diverge, as the delay increases, due to customers leaving the system. These programs model the pure Erlang C relationships and do not compensate for customer abandonments.

PROGRAM NUMBER 1, EXAMPLES:

*G
WHAT IS THE NUMBER OF SERVERS?:10
WHAT IS THE LOAD IN ERLANGS?:9.5
THE ERLANG C BLOCKING PROBABILITY IS: = 0.8255850

*G
WHAT IS THE NUMBER OF SERVERS?:10
WHAT IS THE LOAD IN ERLANGS?:8.0
THE ERLANG C BLOCKING PROBABILITY IS: = 0.4091800

G
WHAT IS THE NUMBER OF SERVERS?:35
WHAT IS THE LOAD IN ERLANGS?:32.2
THE ERLANG C BLOCKING PROBABILITY IS: = 0.5285610

G
WHAT IS THE NUMBER OF SERVERS?:60
WHAT IS THE LOAD IN ERLANGS?:55.2
THE ERLANG C BLOCKING PROBABILITY IS: = 0.4203840

PROGRAM NUMBER 3, EXAMPLES:

G

WHAT IS THE NUMBER OF SERVERS? :10

WHAT IS THE LOAD IN ERLANGS? :9.5

WHAT ARE THE TOTAL NUMBER OF CALLS? :380

ERLANG C BLOCKING PROBABILITY = 0.8255850

AVERAGE DELAY IN SECONDS = 148.60500

AVERAGE CALL HOLDING TIME = 90.000000 SECONDS

THE DELAY GREATER THAN 10 SECONDS IS: = 0.7809700

THE PER CENT DELAYED GREATER THAN 10 SECONDS IS: = 78.097000

G

WHAT IS THE NUMBER OF SERVERS? :10

WHAT IS THE LOAD IN ERLANGS? :8.0

WHAT ARE THE TOTAL NUMBER OF CALLS? :320

ERLANG C BLOCKING PROBABILITY = 0.4091800

AVERAGE DELAY IN SECONDS = 18.413100

AVERAGE CALL HOLDING TIME = 90.000000 SECONDS

THE DELAY GREATER THAN 10 SECONDS IS: = 0.3276460

THE PER CENT DELAYED GREATER THAN 10 SECONDS IS: = 32.764600

G

WHAT IS THE NUMBER OF SERVERS? :35

WHAT IS THE LOAD IN ERLANGS? :32.2

WHAT ARE THE TOTAL NUMBER OF CALLS? :1656

ERLANG C BLOCKING PROBABILITY = 0.5285610

AVERAGE DELAY IN SECONDS = 13.214000

AVERAGE CALL HOLDING TIME = 70.000000 SECONDS

THE DELAY GREATER THAN 10 SECONDS IS: = 0.3543040

THE PER CENT DELAYED GREATER THAN 10 SECONDS IS: = 35.430400

FIGURE 3

PROGRAM NUMBER 1

C-FOCAL, 1969

```
01.11 C THIS PROGRAM COMPUTES C(C,A) WHICH IS THE ERLANG C
01.12 C      BLOCKING PROBABILITY.
01.13 A "WHAT IS THE NUMBER OF SERVERS?",C,!
01.14 A "WHAT IS THE LOAD IN ERLANGS?",A,!
01.15 IF (2-C)2.11,2.11,5.11

02.11 S B=1./A
02.12 S CINT=FITR(C)
02.13 F D=2.,CINT; DO 6
02.14 GOTO 3.11

03.11 S ERLC=1./(1.+(1.-A/C)*B)
03.13 IF (C-A)4.12,4.11,4.11

04.11 T Z8.28 "THE ERLANG C BLOCKING PROBABILITY IS: ",ERLC,!!: Q
04.12 T " LOAD EXCEEDS SERVERS "; Q

05.11 S ERLC =A
05.12 GOTO 4.11

06.11 S B=(D/A)*(B+1)
```

PROGRAM NUMBER 2

C-FOCAL, 1969

01.11 C THIS PROGRAM COMPUTES THE AVERAGE DELAY (BAR D) IN SECONDS
 01.12 C USING C(C,A), THE ERLANG C BLOCKING PROBABILITY.
 01.13 A "WHAT IS THE NUMBER OF SERVERS?", C, !
 01.14 A "WHAT IS THE LOAD IN ERLANGS?", A, !
 01.15 A "WHAT ARE THE TOTAL NUMBER OF CALLS?", TC, !
 01.16 IF (2-C)2.11, 2.11, 7.11

02.11 S B=1./A
 02.12 S CINT=FITR(C)
 02.13 F D=2., CINT; DO 6
 02.14 GOTO 3.11

03.11 S ERLC=1./(1.+(1.-A/C)*B)
 03.12 S H=A*3600./TC
 03.13 IF (C-A)4.14, 4.11, 3.14
 03.14 S AVE=ERLC*(H/(C-A))

04.11 T 78.08 "ERLANG C BLOCKING PROBABILITY ", ERLC, !!
 04.12 T "AVERAGE DELAY IN SECONDS ", AVE, !!
 04.13 T "AVERAGE CALL HOLDING TIME ", H, " SECONDS", !!; Q
 04.14 T "LOAD EXCEEDS SERVERS"; Q

06.11 S B=(D/A)*(3+1)

07.11 S ERLC=A
 07.12 GOTO 3.12

08.11 C THE FORMULA IS GIVEN AS:
 08.12 C AVERAGE DELAY(BAR D)=C(C,A) * H/(C-A)
 08.13 C C = NUMBER OF SERVERS
 08.14 C A = OFFERED LOAD IN ERLANGS
 08.15 C H = AVERAGE HOLDING TIME
 08.16 C C(C,A) = ERLANG C BLOCKING PROBABILITY

PROGRAM NUMBER 3

C-FOCAL,1969

```

01.11 C THIS PROGRAM COMPUTES THE AVERAGE DELAY (BAR D) IN SECONDS
01.12 C AND THE PERCENT DELAYED GREATER THAN 10 SECONDS,
01.13 C USING C(C,A) THE ERLANG C BLOCKING PROBABILITY.
01.14 A "WHAT IS THE NUMBER OF SERVERS? ",C,!
01.15 A "WHAT IS THE LOAD IN ERLANGS? ",A,!
01.16 A "WHAT ARE THE TOTAL NUMBER OF CALLS? ",TC,!
01.17 IF (2-C)2.11,2.11,7.11

02.11 S B=1./A
02.12 S CINT=FITR(C)
02.13 F D=2.,CINT; DO 6
02.14 GOTO 3.11

03.11 S ERLC=1./(1.+(1.-A/C)*B)
03.12 S H=A*3600./TC
03.13 IF (C-A)4.17,4.18,3.14
03.14 S AVE=ERLC*(H/(C-A))
03.15 S PD=ERLC*FEXP(-(C-A)*10./H)
03.16 S PPD=PD*100.

04.11 T %8.4 "ERLANG C BLOCKING PROBABILITY ",ERLC,!!
04.12 T "AVERAGE DELAY IN SECONDS ",AVE,!!
04.13 T "AVERAGE CALL HOLDING TIME ",H," SECONDS",!!
04.14 T "THE DELAY GREATER THAN 10 SECONDS IS: ",PD,!!
04.15 T "THE PER CENT DELAYED GREATER THAN 10 SECONDS IS: ",PPD,!
04.16 Q
04.17 F "LOAD EXCEEDS SERVERS "; Q
04.18 T "THE ERLANG C BLOCKING PROBABILITY IS: ",ERLC,; Q

06.11 S B=(D/A)*(B+1)

07.11 S ERLC=A
07.12 GOTO 4.18

```