# DECUS
## PROGRAM LIBRARY

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| DECUS NO. | FOCAL8-155 |
| TITLE | FACTORS |
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| SOURCE LANGUAGE | FOCAL-'69 |

# FACTORS

DECUS Program Library Write-up

DECUS NO. FOCAL8-155

This program will do the following:

1. Calculate the prime factorization of a number ,x, and print it in the form:

$$a_1 \uparrow b_1$$
$$a_2 \uparrow b_2$$
$$a_3 \uparrow b_3$$
$$\vdots$$

where $a_n$ is a prime and $b_n$ is its exponent. If X is prime, the program will print: X IS PRIME and quit.

2. Print the number of primes, number of prime factors, and total number of factors.

3. Print Ø (X). This is the number of numbers less than X with no prime factor in common with X. Thus, Ø(3)=2, as neither 1 nor 2 has a prime factor in common with 3. Similarly, Ø(4)=2, Ø(5)=4, Ø(6)=2, etc.

4. Print the sum of all factors, including 1 and X, and also print the sum of non-X factors.

5. Test whether X is a perfect square. If it is, ($\sqrt{X}$) SQUARED is printed out, e.g. for 144 12 SQUARED is printed. If X is not a perfect square, nothing is printed.

6. Test to find if X is abundant, perfect, or deficient. An abundant number is one whose sum of non-X factors is greater than X. A perfect number's non-X factor sum equals X. A deficient number's non-X factor sum is less than X. If X is abundant, the program will test to see if it is multiply perfect. A multiply perfect number's non-X factor sum is a whole multiple of X. For example, the sum of factors for 120 is 360. Thus, the non-X factor sum is 240, and 120 is said to be multiply perfect of class 2.

7. If desired, the program will test to see if X is an amicable number. An amicable number is one whose non-X factor sum equals a number whose non-X factor sum equals X. These numbers, X and its non-X factor sum, are generally referred to as an amicable pair, since the non-X factor sum of either equals the other.

8. If desired, a complete factor list will be printed, two factors per line. Each line contains two factors which equal X when multiplied together. This means, that for a perfect square, the last line contains the square root twice. This could be programmed out,

but was left for uniformity with the other lines. The printout for 144 is:

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|---|---|
| 1 | 144 |
| 2 | 72 |
| 3 | 48 |
| 4 | 36 |
| 6 | 24 |
| 8 | 18 |
| 9 | 16 |
| 12 | 12 |

NOTE: In all cases, the printout is formatted nicely, but it looks better if the equal sign preceding the number being printed is removed. This may be done in FOCAL '69 by replacing the 4551 in location 6002 with a 7600.

NOTE: In 4K FOCAL, the extended functions must be removed prior to loading this program.

NOTE: I suggest that numbers be limited to 6 digits, this being all the accuracy FOCAL can handle. Having had no experience with the FOCAL FOUR-WORD OVERLAY, I couldn't suggest anything here. Probably you could handle 10-digit numbers. If so, change line 9.21 to allow for increased accuracy.

NOTE: I believe that this program could be run on FOCAL 5/69 with little change, as I have attempted to define all variables prior to use. The ASK in sections 10 and 11 would have to be changed. Preceding the A"(Y OR N)"N, place an O C, and after it, an O 1. Change the 14 in the following IF's to a 206 and the 25 to a 217. That should fix it. Note that in FOCAL 5/69, there is no equal preceding numerical output. For frequent use in 5/69, make an overlay changing the lines.

To run:

E A (or H, in FOCAL 5/69)
Load program (Extended functions out)
G (After changes, for 5/69)
X: will be printed
TYPE X, return
Wait. For a very large number, especially a prime, could take quite a while
Output will be printed. When AMICABILITY TEST? (Y OR N): or FACTOR LIST? (Y OR N): is printed, after the colon type a Y for Yes or an N for No. Answering NO to both will cause program to QUIT

Methods used:

To express X as a product of primes, $a_1^{b_1} a_2^{b_2}$ the following method is used:

The initial value of X is set to what is inputted in the first ask. The first divisor, S is set to equal 2. X is tested: If the FITR (XIS) = XIS, S is a divisor and A(I) is set to S and B(I) is set to 1. X is set to equal XIS. The process is again tried. If S is again a divisor, B(I) is

incremented. If it is not a divisor, S is set to 3 and X is again tested, this time using A(2) and B(2). After S is 3, 5, 7, 9, 11, 13, 15... etc. are used, until S is greater than the square root of the present X. When S is greater than the square root, A(N) is set to X and B(N) to 1, as X is prime. If no divisors are found by this method, X is Prime. After this, A(1) and B(1) are printed, then A(2) and B(2), these variables now containing the primes and corresponding exponents.

Number of factors:

$$N = (b_1 + 1)\ (b_2 + 1)\ (b_3 + 1) \ldots$$

Phi:

$$\phi(X) = X\ (1 - {}^{1}/_{a_1})\ (1 - {}^{1}/_{a_2}) \ldots$$

Sum of factors:

$$S = (a_1^{b1} + a_1^{b1-1} \ldots a^{1} + 1)\ (a_2^{b2} + a_2^{b2-1} \ldots a_2^{1} + 1) \ldots$$

Amicability:

The non-X factor sum is cycled through section 2, where its prime factorization is found, and section 7, where its factor sum is found. Its non-X factor sum is then compared to the original X. If they are equal, the numbers are amicable.

Numbers to try:

Perfect Numbers:

6, 28, 496, 8128

Multiply perfect numbers:

120, 672

Amicable pairs:

220, 284
1210, 1184
17, 296, 18, 416

First odd abundant number:

945

Powers of numbers
Prime Test
Factorials
Multiples of Abundant, Perfect, or Deficient Numbers