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DECUS NO.

FOCAL8-118

TITLE

THREE MATHEMATICAL ROUTINES
1. To Raise A + B * I to the N Power
2. Complex Roots of Real Interpreters
3. Cube Route Finder

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SOURCE LANGUAGE

FOCAL

THREE MATHEMATICAL ROUTINES

DECUS Program Library Write-up

DECUS No. FOCAL8-118

TITLE: To Raise a + bi to the Nth Power

ABSTRACT

This program is based on De Moivre's Theorem for raising complex numbers to a given power N. It works with all integral values of A and B and for A and/or B equal to zero. It seems reasonable to assume that it would also work with decimal fractions for A or B. In the illustrative examples the = sign has been deleted to improve the printout of the answers which appear in rectangular coordinate form rather than the trigonometric form.

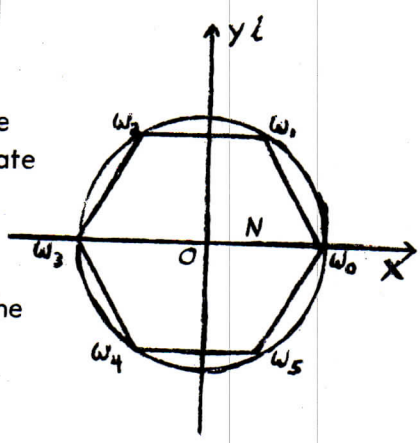

```
*W A
C-FOCAL,1969

01.01 C-TO RAISE  A+B*I  TO THE N POWER
01.02 E
01.05 S PI=3.14159
01.07 A ?A B N?,!
01.14 I (A) 1.2,1.3,1.2
01.20 S TH=FATN[FABS<B/A>]
01.30 I (A) 1.4,1.6,1.5
01.40 I (B) 1.41,1.41,1.43
01.41 S TH=PI+TH; G 2.1
01.43 S TH=PI-TH; G 2.1
01.50 I (B) 1.51,2.1,2.1
01.51 S TH=2*PI-TH; G 2.1
01.60 I (B) 1.63,7.1
01.61 S TH=PI/2; G 2.1
01.63 S TH=3*PI/2

02.10 S Z=FEXP[FLOG<FSQT(A↑2+B↑2)>*N]
02.20 S A=Z*FCOS(TH*N)
02.30 S B=Z*FSIN(TH*N)
02.40 T "(A+B*I)↑N=",A," +",B,"*I",!!; G 1.02

07.10 T "THAT IS THE ORIGIN, 0+0*I!",!
07.20 G 1.02
*Q
```

```
*
*GO
A :1  B :1  N:2

(A+B*I)↑N=    0.000 +    2.000*I

A :1  B :-1  N:3

(A+B*I)↑N=-   2.000 +-   2.000*I

A :-1  B :-1  N:4

(A+B*I)↑N=-   4.000 +    0.000*I

A :-1  B :1.73  N:6

(A+B*I)↑N=   63.659 +    0.195*I

A :3  B :-2  N:3

(A+B*I)↑N=-   9.001 +-  46.000*I

A :0  B :1  N:2

(A+B*I)↑N=-   1.000 +    0.000*I

A :0  B :1  N:3

(A+B*I)↑N=-   0.000 +-   1.000*I

A :0  B :1  N:4

(A+B*I)↑N=    1.000 +-   0.000*I

A :0  B :1  N:5

(A+B*I)↑N=    0.000 +    1.000*I
```

TITLE: To Find the P Complex Roots of a Real Number N

ABSTRACT

This program was planned around the geometrical method of finding the cube roots of unity. The figure shows the location of the 6 roots of N at the vertices of the regular polygon inscribed in the circle. It amounts to sholving an equation $X^P - N = O$. The plane is that of the complex numbers. The printout gives the roots in rectangular coordinate form of the complex number. The results are quite accurate even to having the proper signs for the real and imaginary parts of the roots, depending on which quadrants the radius vectors lie in. The number of roots depends on the value of P, thus meeting the requirements of the Fundamental Theorem of Algebra.




```
*W A
C-FOCAL,1969

01.05 C COMPLEX ROOTS OF REAL INTEGERS
01.10 T !,"X↑P-N=0",!
01.15 E
01.20 A ?P N?,!
01.21 S IM=FLOG(FABS(N))/P
01.22 S N=FEXP(IM)*FSGN(N)
01.40 F I=0,P-1; D 2
01.50 F I=0,P-1; D 3
01.60 T !!; G 1.2

02.10 S X(I)=N*FCOS(I*2*3.14159/P)
02.20 S Y(I)=N*FSIN(I*2*3.14159/P)

03.10 T %1,"W",I,"=",%6.03,X(I)," +",Y(I),"*I",!
*
*
*GO

X↑P-N=0
P :3 N:8

W 0=    2.000 +    0.000*I
W 1=-   1.000 +    1.732*I
W 2=-   1.000 +-   1.732*I


P :3 N:-8

W 0=-   2.000 +    0.000*I
W 1=    1.000 +-   1.732*I
W 2=    1.000 +    1.732*I
```

```
P :4 N:16

W 0=   2.000 +   0.000*I
W 1=   0.000 +   2.000*I
W 2=-  2.000 +   0.000*I
W 3=-  0.000 +-  2.000*I


P :4 N:-16

W 0=-  2.000 +   0.000*I
W 1=-  0.000 +-  2.000*I
W 2=   2.000 +-  0.000*I
W 3=   0.000 +   2.000*I


P :5 N:32

W 0=   2.000 +   0.000*I
W 1=   0.618 +   1.902*I
W 2=-  1.618 +   1.176*I
W 3=-  1.618 +-  1.176*I
W 4=   0.618 +-  1.902*I


P :5 N:-32

W 0=-  2.000 +   0.000*I
W 1=-  0.618 +-  1.902*I
W 2=   1.618 +-  1.176*I
W 3=   1.618 +   1.176*I
W 4=-  0.618 +   1.902*I
```

TITLE: Cube Root Finder

This program gives a very good approximation of the cube root of real numbers. The algorithm is somewhat similar to that for finding square root. A very important step is that in line 3.07. Without it the roots would be far from accurate.

```
*W A
C-FOCAL,1969

01.01 E
01.10 C CUBE ROOT FINDER; T !!

03.01 A ?X?
03.03 S Y=FABS(X)
03.05 S A=FSQT(Y)/2
03.07 F I=1,10; D 4
03.08 G 3.13
03.13 T "CUBE ROOT OF X",FSGN(X)*A; T !!
03.14 G 3.01

04.05 S B=Y/A↑2
04.06 S A=(2*A+B)/3
04.07 R
*
*
*GO
X:27 CUBE ROOT OF X=  3.0000

X:-27 CUBE ROOT OF X=- 3.0000

X:.027 CUBE ROOT OF X=  0.3000

X:100 CUBE ROOT OF X=  4.6416

X:1000 CUBE ROOT OF X= 10.0000

X:10000 CUBE ROOT OF X= 21.5444

X:.125 CUBE ROOT OF X=  0.5000

X:12345 CUBE ROOT OF X= 23.1116
```