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DECUS NO.	8-134
TITLE	LSQ (LEAST SQUARES SUBROUTINE)
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SOURCE LANGUAGE	

LSQ (LEAST SQUARES SUBROUTINE)

DECUS Program Library Write-up DECUS No. 8-134

This subroutine calculates the slope and intercept for

$$y_i = mx_i + b$$

by the method of least squares. The rms deviation of y, the correlation coefficient and a estimate of the percent error in the slope are also calculated.

Specifications: The program requires 1.5 pages of memory and the following page 0 locations:

10-13 temporary storage
100-113;156-160 data return from subroutine
117-123 data for subroutine
114-116;153-155;161-177 temporary storage

The page zero locations may be changed as desired by changing the list which preceeds the program on the symbolic tape. The subroutine calls the subroutine FLOAT, and uses the floating point interpreter, version C.

Description: Before calling the subroutine, the following locations are set up:

117 N = the number of data points
120 LX = starting location for X array.
121 LY = starting location for Y array
122 LCALC = starting location of YCALC array into which the calculated values of y are to be placed
123 LDIF = starting location of DIF array into which the differences between Y and YCALC are to be placed.

The subroutine is called by an effective JMS I LSQ. On return to the calling program, the following will be available:

100	S = slope
103	B = intercept
106	R = correlation coefficient
111	ERROR = rms deviation in y
156	TEM2 = % error in slope

Method of Calculation :

The slope and intercept are obtained using the usual expressions:

SLOPE = (SUMX*SUMY - G*SUMXY)/(SUMX**2 - G*SUMXX)

B = (SUMX*SUMXY - SUMY*SUMXX)/(SUMX**2 - G*SUMXX)

where G is the number of points in floating point form. The correlation coefficient is calculated from:

R = (SUMX*SUMY - G*SUMXY)/SQRT((SUMY**2-G*SUMYY)*(SUMX**2-G*SUMXX))

and the % error in the slope is estimated from:

$$\%ERROR = G*SQRT(SUM(((X_i-\bar{X})*DIF_i)**2))/(SUMX*SUMY - G*SUMXY)$$

The quantity $SUM(((X_i-\bar{X})*DIF_i)**2)$ is collected as SUMXY on pg. 2 of the subroutine. SUMX is set equal to $\bar{X}$, and TEM3 contains the numerator from the slope calculation. The above method of estimating the error in slope makes the error independent of translation of the x axis and was suggested by E. C. Lupton, Jr.

```
                /LEAST SQUARES SUBROUTINE
                IX=10
                IY=11
                ICALC=12
                IDIF=13
                S=100
                B=103
                R=106
                ERROR=111
                G=114
                N=117
                LX=120
                LY=121
                LCALC=122
                LDIF=123
                SUMYY=153
                TEM2=156
                TEM3=161
                SUMX=164
                SUMY=167
                SUMXY=172
                SUMXX=175
                CALL=4400
                RETURN=5400
                FLOAT=4355
                FSQRT=2
                FLPT=4407
                *3400
3400  0000  LSQ,       0
3401  7200            CLA
3402  1117            TAD N
3403  7041            CIA
3404  3366            DCA NEGN              /INDEX
3405  1120            TAD LX                /SET UP REGISTERS
3406  3010            DCA IX
3407  1121            TAD LY
3410  3011            DCA IY
3411  4407            FLPT
3412  5361            FGET ZERO
3413  6164            FPUT SUMX
3414  6167            FPUT SUMY
3415  6175            FPUT SUMXX
3416  6172            FPUT SUMXY
3417  6153            FPUT SUMYY
3420  0000            FEXT
3421  4407  BEGIN,    FLPT
3422  5167            FGET SUMY
3423  1411            FADD I IY
3424  6167            FPUT SUMY             /STORE SUM Y
3425  5410            FGET I IX             /GET X
3426  1164            FADD SUMX
3427  6164            FPUT SUMX             /STORE SUM X
3430  5410            FGET I IX
3431  3410            FMPY I IX
3432  1175            FADD SUMXX
```

```
3433  6175          FPUT SUMXX          /STORE SUM X*X
3434  5410          FGET I IX
3435  3411          FMPY I IY
3436  1172          FADD SUMXY
3437  6172          FPUT SUMXY          /STORE SUM X*Y
3440  5411          FGET I IY
3441  3411          FMPY I IY
3442  1153          FADD SUMYY
3443  6153          FPUT SUMYY          /STORE SUM Y*Y
3444  0000          FEXT                /FINISHED SUMMATIONS
3445  2011          ISZ IY
3446  2011          ISZ IY
3447  2011          ISZ IY
3450  2010          ISZ IX
3451  2010          ISZ IX
3452  2010          ISZ IX
3453  2366          ISZ NEGN
3454  5221          JMP BEGIN           /CONTINUE SUMMATION LOOP
3455  7200          CLA
3456  1117          TAD N               /FLOAT N
3457  4777          CALL FLOAT
3460  4407          FLPT
3461  6114          FPUT G
3462  5175          FGET SUMXX          /CALC S AND B
3463  3114          FMPY G
3464  6156          FPUT TEM2
3465  5164          FGET SUMX
3466  3164          FMPY SUMX
3467  2156          FSUB TEM2
3470  6156          FPUT TEM2           /DENOMINATOR
3471  5172          FGET SUMXY
3472  3114          FMPY G
3473  6161          FPUT TEM3
3474  5164          FGET SUMX
3475  3167          FMPY SUMY
3476  2161          FSUB TEM3
3477  6111          FPUT ERROR          /NUMERATOR, TEMP STORAGE
3500  4156          FDIV TEM2           /SLOPE
3501  6100          FPUT S
3502  5175          FGET SUMXX
3503  3167          FMPY SUMY
3504  6161          FPUT TEM3
3505  5164          FGET SUMX
3506  3172          FMPY SUMXY
3507  2161          FSUB TEM3
3510  4156          FDIV TEM2
3511  6103          FPUT B              /INTERCEPT
3512  5153          FGET SUMYY          /CALCULATE R
3513  3114          FMPY G
3514  6161          FPUT TEM3
3515  5167          FGET SUMY
3516  3167          FMPY SUMY
3517  2161          FSUB TEM3
3520  3156          FMPY TEM2
3521  0002          FSQRT
```

```
3522  6161            FPUT TEM3
3523  5111            FGET ERROR
3524  4161            FDIV TEM3
3525  6106            FPUT R
3526  0000            FEXT
3527  7200            CLA                     /SET UP REGISTERS
3530  1120            TAD LX
3531  3010            DCA IX
3532  1121            TAD LY
3533  3011            DCA IY
3534  1122            TAD LCALC
3535  3012            DCA ICALC
3536  1123            TAD LDIF
3537  3013            DCA IDIF
3540  1117            TAD N
3541  7041            CIA
3542  3776            DCA NEGN2               /INDEX
3543  4407            FLPT
3544  5111            FGET ERROR
3545  6161            FPUT TEM3               /NUMERATOR
3546  5361            FGET ZERO
3547  6111            FPUT ERROR
3550  6172            FPUT SUMXY
3551  6167            FPUT SUMY               /ERROR SUMS
3552  0000            FEXT
3553  4755            CALL CONT
3554  5600            RETURN LSQ
3555  3600  CONT,      3600
3556  0000  ONE,       FLTG 1.0
3557  3777
3560  7774
3561  0000  ZERO,      FLTG 0.0
3562  0000
3563  0000
3564  7777  M1,        -1
3565  7776  M2,        -2
3566  0000  NEGN,      0
            /PG 2 OF LEAST SQUARES PROGRAM
3576  3666  *3600
3577  4355
3600  0000  LSQ2,      0
3601  4407            FLPT
3602  5164            FGET SUMX
3603  4114            FDIV G
3604  6164            FPUT SUMX
3605  0000            FEXT
3606  4407  LOOP,      FLPT
3607  5410            FGET I IX               /CALC RMS ERROR
3610  3100            FMPY S                  /AND ERROR IN SLOPE
3611  1103            FADD B
3612  6412            FPUT I ICALC
3613  2411            FSUB I IY
3614  6413            FPUT I IDIF
3615  3413            FMPY I IDIF
3616  1111            FADD ERROR
```

```
3617  6111              FPUT ERROR            /SUM (ERROR)**2
3620  5410              FGET I IX
3621  2164              FSUB SUMX
3622  3413              FMPY I IDIF
3623  6156              FPUT TEM2
3624  3156              FMPY TEM2
3625  1172              FADD SUMXY
3626  6172              FPUT SUMXY            /SUM (X-XAVE)*(ERROR)**2
3627  0000              FEXT
3630  2010              ISZ IX
3631  2010              ISZ IX
3632  2010              ISZ IX
3633  2011              ISZ IY
3634  2011              ISZ IY
3635  2011              ISZ IY
3636  2012              ISZ ICALC
3637  2012              ISZ ICALC
3640  2012              ISZ ICALC
3641  2013              ISZ IDIF
3642  2013              ISZ IDIF
3643  2013              ISZ IDIF
3644  2266              ISZ NEGN2
3645  5206              JMP LOOP
3646  4407              FLPT
3647  5111              FGET ERROR
3650  4114              FDIV G
3651  0002              FSQRT
3652  6111              FPUT ERROR            /ERROR CALCULATED
3653  5172              FGET SUMXY
3654  0002              FSQRT
3655  3114              FMPY G
3656  4161              FDIV TEM3
3657  3263              FMPY HUNDRED
3660  6156              FPUT TEM2             /% ERROR IN SLOPE
3661  0000              FEXT
3662  5600              RETURN LSQ2
3663  0007  HUNDRED,    FLTG 100.0
3664  3077
3665  7776
3666  0000  NEGN2,      0
```

```
                /SUBROUTINE TO FLOAT FIXED POINT NUMBERS
                /NUMBER IS IN AC; RETURNED IN FL(AC)
                *4355
4355  0000      FLOAT,    0
4356  3045                DCA 45              /PUT NUMBER IN HIGH ORDER
4357  3046                DCA 46              /PUT ZERO IN LOW ORDER
4360  1366                TAD C13             /11(10) INTO
4361  3044      C,        DCA 44              /EXPONENT
4362  4407                JMS I 7             /CALL INTERPRETER
4363  7000                FNOR                /NORMALIZE
4364  0000                FEXT                /LEAVE INTERPRETER
4365  5755                JMP I FLOAT
4366  0013      C13,      13                  /11 DECIMAL
```

(Cf. Floating-Point System Programming Manual p 3-10)